**ST. MARY'S COLLEGE (Autonomous)**

*(Re-accredited with 'A' Grade by NAAC)*

**Thoothukudi-628001, Tamil Nadu**

**(Affiliated to Manonmaniam Sundaranar University)**


# Syllabus ( w.e.f. 2018 )

# M.Phil Mathematics

**Vision**

To empower students with sound knowledge and investigate new methodologies for future applications. The department aims to be a center of excellence in mathematics and computing and is vigorously engaged in both research and teaching.

**Mission**

- To impart the knowledge of mathematical science with precision
- To motivate the students to pursue research in various fields
- To generate mathematical thinking to solve complex problems by providing mathematical experiences, exploring and learning opportunities, and relevant applications in the study and understanding of operations and measurements, as expressed by numbers and symbols.

| PO No. | At the end of the M.Phil. program, the students will be able to: |
|---|---|
| PO-1 | Train students to conduct field based research studies including selection of research problems, sampling, and preparation of research tools and adoption of statistical methodologies. |
| PO-2 | Motivated to take up research as their career. They will also be trained to write research papers and Ph.D. thesis. |
| PO-3 | Apply knowledge of Mathematics, in all the fields of learning including higher research and its extensions. |
| PO-4 | Demonstrate critical understanding, at an advanced level, of up-to-date knowledge and research methodology of a particular field. |
| PO-5 | Implement effective academic and personal strategies for carrying out research projects independently and ethically |
| PO-6 | Contribute original knowledge in response to issues in their specialist area. |
| PO-7 | Communicate research findings at a diverse range of levels and through a variety of media |
| PO-8 | Evaluate one's own research in relation to important and latest issues in the field |
| PO-9 | Demonstrate flexibility to accommodate new knowledge and perspectives |
| PO-10 | Engage in intellectual exchange with researchers from other disciplines to address important research issues |

## Course Structure (w.e.f 2018)

### SEMESTER I

| Subject | Sub. Code | Title of the Paper | Hours / Week | | | Credits | Max. Marks | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | Theory | Library | Seminar | | CIA | ESE | Total |
| Core I | 18MLMC11 | Research Methodology | 4 | 4 | 2 | 4 | 50 | 50 | 100 |
| Core II | 18MLMC12 | Commutative Algebra | 4 | 4 | 2 | 4 | 50 | 50 | 100 |
| Elective I | 18MLME11 | Banach Algebra and Spectral Theory / Advanced Topology/ Advanced Graph theory | 4 | 4 | 2 | 4 | 50 | 50 | 100 |
| | | **Total** | **30** | | | **12** | **150** | **150** | **300** |

### SEMESTER II

| Subject | Sub. Code | Title | No. of Hours | Credits | Dissertation | | Viva Voce | Total |
|---|---|---|---|---|---|---|---|---|
| | | | | | Internal | External | | |
| Project | 18MLMP21 | Project and viva voce | 30 | 12 | 50 | 25 | 25 | 100 |

**Programme Specific Outcomes**

| PSO No. | Upon completion of the M.Phil. Mathematics program, students will be able to: |
|---|---|
| **PSO-1** | Develop research level thinking in the field of pure and applied mathematics. |
| **PSO-2** | Assimilate complex mathematical ideas and arguments. |
| **PSO-3** | Improve your own learning and performance. |
| **PSO-4** | Develop abstract mathematical thinking. |
| **PSO-5** | Apply mathematical ideas to model real-world problems(applied mathematics). |
| **PSO-6** | Apply rigorous, analytic, highly numerate approach to analyze, execute tasks and solve problems in daily life and at work |
| **PSO-7** | Appreciate the role of mathematical proof as a means of conveying mathematical knowledge |
| **PSO-8** | Use Network Flow theory to solve many real time problem such as, Hall's Marriage problem, Konig's problem, deduction of Menger's theorem, finding maximum matching in bipartite graphs. |
| **PSO-9** | Apply the concepts of graph connectivity at appropriate points of graph theory and identify the surfaces which the given graph can be embedded. |
| **PSO-10** | Build a range of appropriate general skills including IT competency. |

| SEMESTER I | | | |
|---|---|---|---|
| **Core I – Research Methodology** | | | |
| **Code : 18MLMC11** | **Hrs/ week : 4** | **Hrs/Semester : 60** | **Credits : 4** |

**Vision:**

To empower students with sound knowledge and investigate new research methodologies for futureapplications.

**Mission:**

To impart the knowledge of mathematical science precision and motivate the students to pursue research in various fields.

**Course Outcomes:**

| CO.No. | Upon completion of this course, students will be able to | PSOs addressed | CL |
|---|---|---|---|
| CO-1 | Understand some basic concepts of research and its methodologies | 1 | Un |
| CO-2 | Select and define appropriate research problem and parameters | 1,5 | Cr |
| CO-3 | Write a research report and thesis | 1 | Ap |
| CO-4 | Identify appropriate research topics | 1 | Un |
| CO-5 | Understand the fundamental concepts of real analysis and their role in modern mathematics and applied context | 5 | Un |
| CO-6 | Use real analysis techniques accurately and efficiently | 5, 7 | Ap |
| CO-7 | Demonstrate capacity for mathematical reasoning through analyzing, proving and explaining concept from real analysis | 5 , 4 | Un |
| CO-8 | Solve basic problems of fourier theory on the real line and state the axioms of $L_p$spaces on the real line | 3 ,7 | Cr |

| SEMESTER I | | | |
|---|---|---|---|
| **Core I – Research Methodology** | | | |
| **Code : 18MLMC11** | **Hrs/ week : 4** | **Hrs/Semester : 60** | **Credits : 4** |

**Unit I**

Writing language of theorems&Plagiarism: Introduction and Motivation - Mathematical style - Terminology and notation (especially in discrete mathematics) - Englishs usage in mathematical writing. Plagiarism – What is Plagiarism - Types of Plagiarism- Glossary – preventing plagiarism when writing.

**Unit II**

Set–theoretic notations and terminology – The concept of measurability – Simple functions – Elementary properties of measures – Arithmetic in $[0,\infty]$.

**Unit III**

Integration of positive functions – Integration of complex functions – The role played by sets of measure zero.

**Unit IV**

$L^p$ - Spaces : Convex functions and inequalities – The $L^p$ - Spaces – Approximation by continuous functions.

**Unit V**

Fourier Transforms: Formal properties – The inversion theorem – The Plancherel theorem – The Banach algebra $L^1$.

**Books for Reference:**


1. DOUGLAS B. WEST ,*THE GRAMMAR ACCORDING TO WEST* .
2. http://www.plagiarism.org/plagiarism-101/what-is-plagiarism/
3. Walter Rudin - *Real and Complex Analysis*, Third edition 2006 – McGraw Hill Book company.
4. C.R. Kothari , *Research Methodology*, Second Revised Edition, New Age International (P) Limited, Publishers,2009.
5. Walter Rudin, *Functional Analysis*, Second edition ,Tata McGraw Hill Publishing company limited 2006.

| SEMESTER I | | | |
|---|---|---|---|
| **Core II– Commutative Algebra** | | | |
| **Code : 18MLMC12** | **Hrs/ week : 4** | **Hrs/Semester : 60** | **Credits : 4** |

**Vision:**

To acquire thorough knowledge about the main concepts of algebra.

**Mission:**

ToDisplay knowledge and to understand further studies in commutative algebra

**Course Outcomes:**

| CO.No. | Upon completion of this course, students will be able to | PSOs addressed | CL |
|---|---|---|---|
| CO-1 | Define basic concepts and construction in commutative algebra, as ideals of various sorts , modules, tensor products, localization, primary decomposition | 4 | Un |
| CO-2 | Familiar with the basic notions of commutative algebra | 4 | Un |
| CO-3 | Preform simple specific calculations in rings | 4 ,5 | Cr , Un |
| CO-4 | Display knowledge and understanding for further studies in commutative algebra | 7 | Un |
| CO-5 | Use the results in commutative algebra to perform simple reasoning to show properties of rings and modules | 4, 6 | An |
| CO-6 | Acquire knowledge on going down and going up theorems | 3 , 4 | Un |
| CO-7 | State the main theorems about rings and modules | 4 | Cr |
| CO-8 | Understand the concepts and theorems on chain conditions | 4 | Un |

| SEMESTER I | | | |
|---|---|---|---|
| **Core II– Commutative Algebra** | | | |
| **Code : 18MLMC12** | **Hrs/ week : 4** | **Hrs/Semester : 60** | **Credits : 4** |

**Unit I**

Rings and Ideals – Rings and homomorphisms – Ideals-Quotient rings – Zero divisors Nilpotent elements - units – Prime ideals and Maximal ideals – Nilradical and Jacobson radical – Operations on ideals – Extension and Contraction.

**Unit II**

Modules – Modules and module homomorphisms – Submodules and quotient modules – Operations on submodules – Direct sum and product – Finitely generated modules - Exact sequences – Tensor product of modules - Restriction and extension of scalars – Exactness properties of the tensor product.

**Unit III**

Rings and Modules of Fractions - Local properties – Extended and contracted ideals in rings of fractions – Primary Decomposition – Noetherian Rings – Primary decomposition in Noetherian rings.

**Unit IV**

Integral Dependence and Valuations - Integral dependence – The going - up theorem - Integrally closed integral domains - The going - down theorem - valuation rings - Chain Conditions.

**Unit V**

Artin Rings – Structure theorem to Artin rings – Discrete Valuation Rings and Dedekind Domains - Discrete valuation rings – Dedekind domains – Fractional ideals.

**Books for Reference:**

1.M.F. Atiyah and I.G. Macdonald , *Introduction to Commutative Algebra*, Addison-Wesley Publishing Company, 1969.

2. Thomas W. Hungerford, Springer Verlog ,*Algebra* , Indian repaint , 2004.

3. S.N. Gopalakrishnan,Commutative Algebra.

| SEMESTER I | | | |
|---|---|---|---|
| **Elective – Banach Algebra and Spectral Theory** | | | |
| **Code : 18MLME11** | **Hrs/ week : 4** | **Hrs/Semester : 60** | **Credits : 4** |

**Vision:**

To discuss the applications of spectral theory in banach algebra and give some examples

**Mission:**

To establish some results on the boundary spectrum and investigatesome properties and its relations with a spectral radius

**Course Outcomes:**

| CO.No. | Upon completion of this course, students will be able to | PSOs addressed | CL |
|---|---|---|---|
| CO-1 | Discuss about the complex homomorphism and the basic properties of spectra | 2 | An |
| CO-2 | Understand the concept of group of invertible elements, ideals, homomorphisms and Gelfand transforms. | 4 | Un |
| CO-3 | Understand how banach space and banach algebra theory is useful in concrete situations in analysis. | 6 | Cr , Un |
| CO-4 | Understand how the number of concepts in banach space theory are developed as tools to compensate the lack of an inner products in hilbert spaces. | 7, 6, 4 | Un |
| CO-5 | Understand how algebra and analysis contribute to a theory which would have a much more technical character when only one of the buildings blocks where used. | 3,4 | An |
| CO-6 | Acquire the concepts of Involutions , Applications to non commutative algebras and Positive functionals. | 3 and 4 | Ap |
| CO-7 | Understand the concepts of Bounded operators and able to prove spectral theorem | 7 | Un,Cr |
| CO-8 | Understand the concepts ofEigen Values of normal operators ,positive operators and square roots ,the group of invertible operators and Characterization of B* - Algebras. | 7 | Un |

| SEMESTER I | | | |
|---|---|---|---|
| **Elective – Banach Algebra and Spectral Theory** | | | |
| **Code : 18MLME11** | **Hrs/ week : 4** | **Hrs/Semester : 60** | **Credits : 4** |

**Unit I**

Introduction – complex homomorphism- basic properties of spectra – symbolic calculus.

**Unit II**

Differentiation - the group of invertible elements – ideals and homomorphisms - Gelfand transforms.

**Unit III**

Involutions – Applications to non commutative algebras– Positive functionals.

**Unit IV**

Bounded operators – Basic facts – Bounded operators – A commutativity theorem – Resolution of the identity – Spectral theorem.

**Unit V**

Eigen Values of normal operators – positive operators and square roots – the group of invertible operators – Characterization of B* - Algebras.

**Book for Reference:**

1. Walter Rudin, *Functional Analysis,* First Edition, Tata McGraw Hill Publishing company Ltd, New Delhi.
2. Casper Goffman and George Pedrick, *First Course in Functional Analysis* - Prentice-Hall of India Private Limited, New Delhi, 1974.

| SEMESTER I | | | |
|---|---|---|---|
| **Elective – Advanced Topology** | | | |
| **Code : 18MLME11** | **Hrs/ week : 4** | **Hrs/Semester : 60** | **Credits : 4** |

**Vision:**

To equip the students with the advanced Research topics in Topology.

**Mission:**

To provide an environment where students can learn advanced topology and became competent uses of mathematics and mathematical application.

**Course Outcome:**

| CO.No. | Upon completion of this course, students will be able to | PSOs addressed | CL |
|---|---|---|---|
| CO-1 | Express $T_1$, $T_2$, $T_3$, $T_4$ separation axioms and use them to prove various properties. | 4 and 5 | Cr |
| CO-2 | Construct the product topology and product spaces. | 4 | Ap |
| CO-3 | Solve challenging problems, develop the proofs of theorems on their own and present those proves clearly. | 3,6 | Cr |
| CO-4 | Define connectedness and compactness and prove a selection of related theorems. | 4 | Ap |
| CO-5 | Describe different examples distinguishing general geometric and bitopology. | 7 | Ap |
| CO-6 | Understand the concept of complete metric spaces. | 3 | Un |
| CO-7 | Understand the concepts of Pointwise and Compact Convergence and able to write the proof of Ascoli's Theorem. | 3 And 4 | Un |
| CO-8 | Understand the various concepts of Bitopological Separation | 4 | Un |

| SEMESTER I | | | |
|---|---|---|---|
| **Elective – Advanced Topology** | | | |
| **Code : 18MLME11** | **Hrs/ week : 4** | **Hrs/Semester : 60** | **Credits : 4** |

**Unit I The Stone-Cech Compactification**

The Stone-Cech Compactification - Local Finiteness.

**Unit II Metrization Theorems and Paracompactness**

The Nagata – Smirnov Metrization Theorem - Paracompactness - The Smirnov Metrization Theorem.

**Unit III Complete Metric Spaces**

Complete Metric Spaces – A Space-Filling Curve - Compactness in Metric Spaces.

**Unit IV Compact Convergence and Baire Spaces**

Pointwise and Compact Convergence - Ascoli's Theorem – Baire Spaces.

**Unit V Bitopological Separation**

Pairwise and Spaces - Pairwise Regular Bitopological Spaces – Quasi-Metrizable Bitopological Spaces - Pairwise Normal Bitopological Spaces- Pairwise Completely Normal Bitopological Spaces.

**Books for Reference:**


1. James R. Munkres-Topology-Second Edition 2006-Prentice-Hall of India Private Ltd.
2. K.Chandrasekhara Rao ,*Topology*, Narosa Publishing House Pvt.Ltd, 2009.
3. G.F. Simmons *Introduction to Topology and Modern Analysis* by Tata McGraw Hill Publishing Company Limited, 2006.
4. Tej Bahadur Singh, *Elements of Topology* , CRC Press , 2013.
5. Sheldon W. Davis , *Topology*, Tata McGraw- Hill Edition 2006.

| SEMESTER I | | | |
|---|---|---|---|
| **Elective – Advanced Graph Theory** | | | |
| **Code : 18MLME11** | **Hrs/week : 4** | **Hrs/Semester : 60** | **Credits : 4** |

**Vision:**

To get comprehensive idea on the recent trends in the topics of Graph theory.

**Mission:**

To acquire a strong background of graph theory which has diverse applications in various fields.

**Course Outcomes:**

| CO.No. | Upon completion of this course, students will be able to | PSOs addressed | CL |
|---|---|---|---|
| CO-1 | Understand the concept of Connectivity and various kinds of labelling. | 9 | Un |
| CO-2 | Equipthe concept of the bounds in terms of order, degree and packing | 8,9,10 | An |
| CO-3 | Understand the concept of region coloring and Nowhere zero k-flow. | 8 , 10 | Un |
| CO-4 | Guarantee the existence of graph of specified type. | 9 | An |
| CO-5 | Study the concept of decompositions of a graph and the relations between complementary graphs and decompositions of several graphs. | 8 , 9 | Re |
| CO-6 | Gain basic ideas about the Hereditary ,Superhereditary, Properties of Independence sets , Dominating sets Irredundant sets andthe Domination Chain. | 8 , 9 | Un |
| CO-7 | Give knowledge about advanced models and methods in graph theory. | 8 | Un |
| CO-8 | Find center and Eccentricity of various graphs and to accquire the knowledge of self-centered graphs. | 8 and 9 | Cr |

| SEMESTER I | | | |
|---|---|---|---|
| **Elective – Advanced Graph Theory** | | | |
| **Code : 18MLME11** | **Hrs/week : 4** | **Hrs/Semester : 60** | **Credits : 4** |

**Unit I Labelling**

Graceful and Related Labelling – Arithmetic and Other Related Labellings – Divisor Graphs.

**Unit II Bounds on the Domination Number**

Bounds in Terms of Order - Bounds in Terms of Order, Degree and Packing - Bounds in Terms of Order and Size - Bounds in Terms of Degree, Diametre and Girth - Bounds in Terms of Independence and Covering - Product Graphs and Vizings's Conjecture.

**Unit III Domination, Independence and Irredundance**

Hereditary and Superhereditary Properties – Independence sets – Dominating sets - Irredundant sets – The Domination Chain – Extensions Using Maximality and Minimality

**Unit IV Decompositions of a Graph**

Nordhaus – Gaddum type theorems – Some relations between complementary graphs and between decompositions of several graphs.

**Unit V Distance in Graphs**

The center and Eccentricity – Self-Centered Graphs – The Median – Central Paths – Other Generalized Centers.

**Books for Reference:**


1. G. Suresh Singh, *Graph Theory*, PHI Learning Private Limited, 2010.
2. Teresa W. Haynes, Stephen T. Hedetniemi, Peter J. Slater *Fundamentals of Domination in Graphs*, Marcel Dekker, INC., 1998.
3. Juraj Bosak, *Decompositions of Graphs*, Kluwer Academic Publishers, 1990.
4. Fred Buckley, Frank Harary, *Distance in Graphs*, Addison Wesley Publishing Company, 1989.
5. Pramima Panigrahi S. B. Rao, *Graph Theory Research Directions*, Narosa Publishing House.
6.. John M. Harris, Jeffry L. Hirst, Michael J. Mossinghof, *Combinatorics and Graph Theory*, Second Edition.