A View on Separation Axioms via Nano $g\alpha^*$- Open Sets

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ABSTRACT: The basic objective of this paper is to define nano generalized α^*-T_0 space, nano generalized α^*-T_1 space, nano generalized α^*-T_2 space and investigate their properties. Also we have obtained some of their basic results and given appropriate examples to understand the abstract concepts clearly.

KEY WORDS: Nano generalized α^*- T_0 space, Nano generalized α^*-T_1 space, Nano generalized α^*- T_2 space.

I INTRODUCTION

The theory of nano topology proposed by Lellis Thivagar [3] and Carmel Richard is an extension of set theory for the study of intelligent systems characterized by insufficient and incomplete information. The elements of a nano topological space are called the nano open sets. The author has defined nano topological space in terms of lower and upper approximations and boundary region. Further, various topological concepts were studied in the subsequent years. The concept of separation axioms in nano topology was introduced by Sathishmohan et.al.[5] P. Anbarasi Rodrigo and P. Subithra [1] have presented a new class of nano generalized closed sets namely nano generalized α^*-closed sets. The aim of this paper is to introduce and study the properties of nano generalized α^*-T_0 , nano generalized α^*-T_1 and nano generalized α^*-T_2 in nano topological spaces.

II PRELIMINARIES

Throughout this paper $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$ represent nano topological spaces on which no separation axioms are assumed unless and otherwise mentioned. For a subset $\mathbb{S}$ of $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$, $Ncl(\mathbb{S})$ and $Nint(\mathbb{S})$ denote the nano closure of $\mathbb{S}$ and nano interior of $\mathbb{S}$ respectively. We recall the following definitions, which are useful in the sequel.

Definition 2.1[3]: Let $\mathbb{U}$ be a non-empty finite set of objects called the universe and $\mathbb{R}$ be an equivalence relation on $\mathbb{U}$ named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(\mathbb{U}, \mathbb{R})$ is said to be the approximation space. Let $\mathbb{X} \subseteq_{\mathbb{N}} \mathbb{U}$. Then

1) The lower approximation of $\mathbb{X}$ with respect to $\mathbb{R}$ is the set of all objects, which can be for certain classified as $\mathbb{X}$ with respect to $\mathbb{R}$ and it is denoted by $\mathbb{L}_{\mathbb{R}}(\mathbb{X})$.

$\mathbb{L}_{\mathbb{R}}(\mathbb{X}) = \cup_{x\in\mathbb{U}}\{\mathbb{R}(x): \mathbb{R}(x) \subseteq_{\mathbb{N}} \mathbb{X}\}$.

2) The upper approximation of $\mathbb{X}$ with respect to $\mathbb{R}$ is the set of all objects, which can be possibly classified as $\mathbb{X}$ with respect to $\mathbb{R}$ and it is denoted by $\mathbb{U}_{\mathbb{R}}(\mathbb{X})$.

$\mathbb{U}_{\mathbb{R}}(\mathbb{X}) = \cup_{x\in\mathbb{U}}\{\mathbb{R}(x): \mathbb{R}(x) \cap \mathbb{X} \neq \phi\}$.

3) The boundary region of $\mathbb{X}$ with respect to $\mathbb{R}$ is the set of all objects, which can be classified neither as $\mathbb{X}$ nor as not –$\mathbb{X}$ with respect to $\mathbb{R}$ and it is denoted by $\mathbb{B}_{\mathbb{R}}(\mathbb{X})$.

$\mathbb{B}_{\mathbb{R}}(\mathbb{X}) = \mathbb{U}_{\mathbb{R}}(\mathbb{X}) - \mathbb{L}_{\mathbb{R}}(\mathbb{X})$.

Definition 2.2[3]: Let $\mathbb{U}$ be the universe, $\mathbb{R}$ be an equivalence relation on $\mathbb{U}$ and $\tau_{\mathbb{R}}(\mathbb{X}) = \{\mathbb{U}, \phi, \mathbb{L}_{\mathbb{R}}(\mathbb{X}), \mathbb{U}_{\mathbb{R}}(\mathbb{X}), \mathbb{B}_{\mathbb{R}}(\mathbb{X})\}$ where $\mathbb{X} \subseteq_{\mathbb{N}} \mathbb{U}$. Then $\tau_{\mathbb{R}}(\mathbb{X})$ satisfies the following axioms:

1) $\mathbb{U}$ and $\phi \in \tau_{\mathbb{R}}(\mathbb{X})$.

2) The union of the elements of any sub collection of $\tau_{\mathbb{R}}(\mathbb{X})$ is in $\tau_{\mathbb{R}}(\mathbb{X})$.

3) The intersection of the elements of any finite sub collection of $\tau_{\mathbb{R}}(\mathbb{X})$is in $\tau_{\mathbb{R}}(\mathbb{X})$. That is, $\tau_{\mathbb{R}}(\mathbb{X})$ is a topology on $\mathbb{U}$ called the nano topology on $\mathbb{U}$ with respect to $\mathbb{X}$. We call $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$ as the nano topological space. The elements of $\tau_{\mathbb{R}}(\mathbb{X})$ are called as nano open sets. The complement of nano-open sets is called nano closed sets.

Definition 2.3[3]: If $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$ is a nano topological space with respect to $\mathbb{X}$ and if $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{U}$, then

(i) The nano interior of $\mathbb{S}$ is defined as the union of all nano open subsets of $\mathbb{S}$ and it is denoted by $\mathbb{N}(\mathbb{S})$. That is, $\mathbb{N}(\mathbb{S})$ is the largest open subset of $\mathbb{S}$.

(ii) The nano closure of $\mathbb{S}$ is define as the intersection of all nano closed sets containing $\mathbb{S}$ and it is denoted by $\mathbb{N}(\mathbb{S})$. That is, $\mathbb{N}(\mathbb{S})$ is the smallest nano closed set containing $\mathbb{S}$.

Definition 2.4[5]: A nano topological space $\mathbb{U}$ is called nano T_0 space for $u, v \in \mathbb{U}$ and $u \neq v$, there exists a nano-open set $\mathbb{G}$ such that $u \in \mathbb{G}$ and $v \notin \mathbb{G}$.

Definition 2.5[5]: A nano topological space $\mathbb{U}$ is called nano T_1 space for $u, v \in \mathbb{U}$ and $u \neq v$, there exists a nano-open sets $\mathbb{G}$ and $\mathbb{H}$ such that $u \in \mathbb{G}$, $v \notin \mathbb{G}$ and $v \in \mathbb{H}$, $u \notin \mathbb{H}$.

Definition 2.6[5]: A nano topological space $\mathbb{U}$ is called nano T_2 space for $u, v \in \mathbb{U}$ and $u \neq v$, there exists disjoint nano open sets $\mathbb{G}$ and $\mathbb{H}$ such that $u \in \mathbb{G}$ and $v \in \mathbb{H}$.

Definition 2.7[1]: A nano generalized α^*-closed set is a subset $\mathbb{S}$ of a $N(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$ if $\mathbb{N}\alpha cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}\mathrm{int}^*(\mathbb{F})$whenever $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F}$ *is* $\mathbb{N}g$-open in $\mathbb{U}$.

Definition 2.8[1]: A subset $\mathbb{S}$ of a $NTS(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$ is called nano generalized α^*-open set if its complement is nano generalized α^*-closed.

Theorem 2.9[1]: Every nano open set is nano generalized α^*-open.

Theorem 2.10[1]: If $\mathbb{G}$ and $\mathbb{H}$ are nano generalized α^*-open sets in nano topological space, then $\mathbb{G} \cup \mathbb{H}$ is a nano generalized α^*-open set.

Definition 2.11[1]: Let $\mathbb{S}$ be a subset of the NTS $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$. Then the intersection of all nano generalized α^*-closed sets containing $\mathbb{S}$ is called nano generalized α^*-closure of $\mathbb{S}$. That is, nano generalized $\alpha^*- cl(\mathbb{S}) = \cap \{\mathbb{R}: \mathbb{R}$ *is* nano generalized $\alpha^* - closed\ sets\ and\ \ \mathbb{S} \subseteq_{\mathbb{N}} \mathbb{R}\}$.

Theorem 2.12[1]: Let $\mathbb{S}$ be a subset of $\mathbb{U}$, then

a) nano generalized $\alpha^*- cl(\mathbb{S})$ is nano generalized α^*-closed in $\mathbb{U}$ and nano generalized $\alpha^*- cl(\mathbb{S})$ is the smallest nano generalized α^*- closed set in $\mathbb{U}$ containing $\mathbb{S}$.

b) $\mathbb{S}$ is nano generalized α^*-closed if and only if nano generalized $\alpha^*- cl(\mathbb{S}) = \mathbb{S}$.

Definition 2.13[2]: A function $f: (\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X})) \rightarrow (\mathbb{V}, \sigma_{\mathbb{R}}(\mathbb{Y}))$ is called Nano generalized α^*-continuous if $\mathbb{f}^{-1}$ $(\mathbb{S})$ is nano generalized α^*-closed in $\left(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X})\right)$ for every nano closed set $\mathbb{S}$ in $(\mathbb{V}, \sigma_{\mathbb{R}}(\mathbb{Y}))$. That is, if the inverse image of every nano closed set in $(\mathbb{V}, \sigma_{\mathbb{R}}(\mathbb{Y}))$ is nano generalized α^*-closed in $\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X})$ then $\mathbb{f}$ is nano generalized α^*- continuous.

Definition 2.14[2]: A function $f: (\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X})) \rightarrow (\mathbb{V}, \sigma_{\mathbb{R}}(\mathbb{Y}))$ is called Nano generalized α^*-irresolute function if the inverse image of every nano generalized α^*-closed set in $(\mathbb{V}, \sigma_{\mathbb{R}}(\mathbb{Y}))$ is nano generalized α^* -closed in $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$.

Remark 2.15: A nano topological space $\mathbb{U}$ is $\mathbf{T_1}$ iff { } is closed in $\mathbb{U}$ for every $u \in \mathbb{U}$.

III NANO GENERALIZED α^*-T_0 SPACES

Definition 3.1: A nano topological space $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$ is said to be nano generalized α^*-T_0 if whenever u and v are distinct points in $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$ there exists a nano generalized α^*-open set in $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$ containing one of u and v but not the other.

Example 3.2: Let $\mathbb{U} = \{p, q, r\}$ with $\mathbb{U}/\mathbb{R} = \{\{p\}, \{p, q\}\}$ and $\mathbb{X} = \{p\} \subseteq_{\mathbb{N}} \mathbb{U}$. Then $\tau_{\mathbb{R}}(\mathbb{X}) = \{\mathbb{U}, \phi, \{p\}, \{q\}, \{p, q\}\}$. Here $\mathbb{N}g\alpha^*$-open$= \{\mathbb{U}, \phi, \{p, q\}, \{q\}, \{p\}\}$. Clearly, it is $\mathbb{N}g\alpha^*$-T_0.

Theorem 3.3: Every Nano T_0 space is $\mathbb{N}g\alpha^*$-T_0 space.

Proof: Let $\mathbb{U}$ be Nano T_0 space, u and v be two distinct points of $\mathbb{U}$. As $\mathbb{U}$ is Nano T_0 there exists a nano-open set $\mathbb{G}$ such that $u \in \mathbb{G}$ and $v \notin \mathbb{G}$, by theorem 2.9, G is $\mathbb{N}g\alpha^*$- open. Hence $\mathbb{U}$ is $\mathbb{N}g\alpha^*$-T_0 space.

Remark 3.4: The converse of the theorem need not be true in general as shown in the following example.

Example 3.5: Let $\mathbb{U} = \{p, q, r, s\}$ with $\mathbb{U}/\mathbb{R} = \{\{p\}, \{r\}, \{q, s\}\}$ and $\mathbb{X} = \{p, q\} \subseteq_{\mathbb{N}} \mathbb{U}$. Then $\tau_{\mathbb{R}}(\mathbb{X}) = \{\mathbb{U}, \phi, \{p\}, \{q, s\}, \{p, q, s\}\}$.

Here $\mathbb{N}g\alpha^*$- open$= \{\mathbb{U}, \phi, \{p, q, s\}, \{q, s\}, \{p, s\}, \{p, q\}, \{s\}, \{q\}, \{p\}\}$. Clearly, it is $\mathbb{N}g\alpha^* T_0$ space. Now, for any two points q and s, there does not exist any Nano open set containing one but not the other. Hence $\mathbb{U}$ is not Nano T_0.

Theorem 3.6: A nano topological space $\mathbb{U}$ is a $\mathbb{N}g\alpha^*$-T_0 if and only if the $\mathbb{N}g\alpha^*$-closure of distinct points are distinct.

Proof: Let u and v be two distinct points of a $\mathbb{N}g\alpha^*$-T_0 space $\mathbb{U}$. We prove that $\mathbb{N}g\alpha^* cl(\{u\}) \neq \mathbb{N}g\alpha^* cl(\{v\})$. Since, $\mathbb{U}$ is $\mathbb{N}g\alpha^*$-T_0, there exists a $\mathbb{N}g\alpha^*$-open set $\mathbb{S}$ in $\mathbb{U}$ such that $u \in \mathbb{S}, v \notin \mathbb{S}$. Also, $u \notin U - \mathbb{S}$ and $v \in U - \mathbb{S}$, where $U - \mathbb{S}$ is $\mathbb{N}g\alpha^*$-closed set in $\mathbb{U}$. Since $\mathbb{N}g\alpha^* cl(\{v\})$ is the intersection of all $Ng\alpha^*$-closed sets which contain v, $v \in \mathbb{N}g\alpha^* cl(\{v\})$ and $u \notin \mathbb{N}g\alpha^* cl(\{v\})$ as $u \notin U - \mathbb{S}$. Therefore, $\mathbb{N}g\alpha^* cl(\{u\}) \neq \mathbb{N}g\alpha^* cl(\{v\})$.

Conversely, suppose the $\mathbb{N}g\alpha^*$-closure of distinct points are different. Let u and v be two distinct points of $\mathbb{U}$. Then $\mathbb{N}g\alpha^* cl(\{u\}) \neq \mathbb{N}g\alpha^* cl(\{v\})$.. Hence there exist a point w in $\mathbb{U}$ such that w lies in only one of the two sets $\mathbb{N}g\alpha^* cl(\{u\})$ and $\mathbb{N}g\alpha^* cl(\{v\})$, say $\mathbb{N}g\alpha^* cl(\{u\})$. If $u \in \mathbb{N}g\alpha^* cl(\{v\})$ then $w \in \mathbb{N}g\alpha^* cl(\{u\}) \subseteq \mathbb{N}g\alpha^* cl(\{v\})$ which implies $w \in \mathbb{N}g\alpha^* cl(\{v\})$ which is a contradiction to $w \notin \mathbb{N}g\alpha^* cl(\{v\})$. Therefore $u \notin \mathbb{N}g\alpha^* cl(\{v\})$.

Hence $\mathbb{U} - \mathbb{N}g\alpha^* cl(\{v\})$ is a $\mathbb{N}g\alpha^*$open set in $\mathbb{U}$ containing u but not v. Hence, $\mathbb{U}$ is $\mathbb{N}g\alpha^*$-T_0 space.

Theorem 3.7: Let $\boldsymbol{f}: (\mathbb{U}, \boldsymbol{\tau}_{\mathbb{R}}(\mathbb{X})) \to (\mathbb{V}, \boldsymbol{\sigma}_{\mathbb{R}}(\mathbb{Y}))$ be a bijection. If f is $\mathbb{N}\boldsymbol{g\alpha}^*$-open and $\mathbb{U}$ **is** $\mathbb{N}\boldsymbol{T_0}$, then $\mathbb{V}$ is $\mathbb{N}\boldsymbol{g\alpha}^*$**-**$\boldsymbol{T_0}$**.**

Proof: Suppose f is $\mathbb{N}g\alpha^*$-open and $\mathbb{U}$ is $\mathbb{N}T_0$. Let $v_1 \neq v_2 \in \mathbb{V}$. Since f is a bijection, there exist u_1, u_2 in $\mathbb{U}$ such that $f(u_1) = v_1$ and $f(u_2) = v_2$ with $u_1 \neq u_2$. Since $\mathbb{U}$ is $\mathbb{N}T_0$, there exists a nano open set $\mathbb{S}$ in $\mathbb{U}$ containing one of u_1 and u_2 but not the other. Since f is $\mathbb{N}g\alpha^*$-open bijection, $(\mathbb{S})$ is a $\mathbb{N}g\alpha^*$-open set in $\mathbb{V}$ containing v_1 or v_2 but not the other. Thus $\mathbb{V}$ is $\mathbb{N}g\alpha^*$-T_0.

Theorem 3.8: Let $f: (\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X})) \to (\mathbb{V}, \sigma_{\mathbb{R}}(\mathbb{Y}))$ be a bijection. If f is $\mathbb{N}g\alpha^*$-continuous and $\mathbb{V}$ is $\mathbb{N}T_0$, then $\mathbb{U}$ is $\mathbb{N}g\alpha^*$-T_0.

Proof:. Suppose f is $\mathbb{N}g\alpha^*$-continuous and $\mathbb{V}$ is $\mathbb{N}T_0$. Let $u_1, u_2 \in \mathbb{U}$ with $u_1 \neq u_2$. Let $v_1 = f(u_1)$ and $v_2 = f(u_2)$. Since f is one to one , $v_1 \neq v_2$. Since $\mathbb{V}$ is $\mathbb{N}T_0$, there exists an nano open set $\mathbb{S}$ in $\mathbb{V}$ containing one of v_1 or v_2 but not the other. Since f is $\mathbb{N}g\alpha^*$- continuous, $f^{-1}(\mathbb{S})$ is a $\mathbb{N}g\alpha^*$-open set in $\mathbb{U}$ containing one of u_1 and u_2 but not the other. Thus $\mathbb{U}$ is $\mathbb{N}g\alpha^*$-T_0.

Theorem 3.9: Let $f: (\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X})) \to (\mathbb{V}, \sigma_{\mathbb{R}}(\mathbb{Y}))$ be a bijection. If f is $\mathbb{N}g\alpha^*$-irresolute and $\mathbb{V}$ is $\mathbb{N}g\alpha^*$-T_0, then $\mathbb{U}$ is $\mathbb{N}g\alpha^*$-T_0.

Proof: Suppose f is $\mathbb{N}g\alpha^*$-irresolute and $\mathbb{V}$ is $\mathbb{N}g\alpha^*$-T_0. Let $u_1, u_2 \in \mathbb{U}$, with $u_1 \neq u_2$. Let $v_1 = f(u_1)$ and $v_2 = f(u_2)$. Since f is one to one, $v_1 \neq v_2$. Since $\mathbb{V}$ is $\mathbb{N}g\alpha^*$-T_0, there exists a $\mathbb{N}g\alpha^*$-open set $\mathbb{S}$ in $\mathbb{V}$ containing one of v_1 or v_2 but not the other. Since f is $\mathbb{N}g\alpha^*$-irresolute, $f^{-1}(\mathbb{S})$ is a $\mathbb{N}g\alpha^*$-open set in $\mathbb{U}$ containing one of u_1 and u_2 but not the other. Thus $\mathbb{U}$ is $\mathbb{N}g\alpha^*$-T_0.

IV NANO GENERALIZED α^*-T_1 SPACES

Definition 4.1: A nano topological space $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$ is said to be nano generalized α^*-T_1 if whenever u and v are distinct points in $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$, there exist nano generalized α^*open sets, one containing u but not v and the other containing v but not u.

Theorem 4.2: For a nano topological space $\mathbb{U}$ the following are equivalent.

(i) $\mathbb{U}$ is a $\mathbb{N}g\alpha^*$-T_1 space.

(ii) for every $u \in \mathbb{U}$, $\{ u \}$ is $\mathbb{N}g\alpha^*$-closed in $\mathbb{U}$.

(iii) Each subset of $\mathbb{U}$ is the intersection of $\mathbb{N}g\alpha^*$-open sets containing it.

(iv) The intersection of all $\mathbb{N}g\alpha^*$-open sets in $\mathbb{U}$ containing the point u is $\{ u \}$.

Proof: (i)⇒(ii) Suppose that $\mathbb{U}$ is $\mathbb{N}g\alpha^*$-T_1. Let $u \in \mathbb{U}$. Then for every $v \neq u$, there exists a $Ng\alpha^*$-open set $\mathbb{G}_v$ in $\mathbb{U}$ containing v but not u. Hence $v \in \mathbb{G}_v \subseteq \mathbb{U} \setminus \{ u \}$.Therefore $\mathbb{U} \setminus \{ u \} = \cup \{ \mathbb{G}_v : v \in \mathbb{U} \setminus \{u\}\}$. By theorem 2.10 $\mathbb{U} \setminus \{ u \}$ is $\mathbb{N}g\alpha^*$-open in $\mathbb{U}$. Therefore $\{ u \}$ is $\mathbb{N}g\alpha^*$-closed.

(ii)⇒(iii) Let $\mathbb{S} \subseteq \mathbb{U}$. Then, for each $u \in \mathbb{U} \setminus \mathbb{S}$, $\{ u \}$ is $\mathbb{N}g\alpha^*$-closed in $\mathbb{U}$ and hence, $\mathbb{U} \setminus \{ u \}$ is $\mathbb{N}g\alpha^*$-open. Clearly, $\mathbb{S} \subseteq \mathbb{U} \setminus \{ u \}$ for each $u \in \mathbb{U} \setminus \mathbb{S}$. Therefore, $\mathbb{S} \subseteq \cap \{ \mathbb{U} \setminus \{ u \} : u \in \mathbb{U} \setminus \mathbb{S} \}$. On the other hand, if $v \notin \mathbb{S}$, then $v \in \mathbb{U} \setminus \mathbb{S}$ and $v \notin \mathbb{U} \setminus \{ v \}$. This implies $v \notin \cap \{ \mathbb{U} \setminus \{ u \} : u \in \mathbb{U} \setminus \mathbb{S} \}$. Hence, $\cap \{ \setminus \{ u \} : u \in \mathbb{U} \setminus \mathbb{S} \} \subseteq \mathbb{S}$. Therefore, $\mathbb{S} = \cap \{ \mathbb{U} \setminus \{ u \} : u \in \mathbb{U} \setminus \mathbb{S} \}$ which proves (iii).

(iii)⇒(iv) Taking $\mathbb{S} = \{ u \}$, by (iii) $\mathbb{S} = \{ u \} = \cap \{ \mathbb{G} : \mathbb{G}$ is $\mathbb{N}g\alpha^*$-open and $u \in \mathbb{G} \}$. This proves (iv).

(iv)⇒(i) Let $u, v \in \mathbb{U}$ with $v \neq u$. Then $v \notin \{ u \} = \cap \{ \mathbb{G} : \mathbb{G}$ is $\mathbb{N}g\alpha^*$-open and $u \in \mathbb{G} \}$. Hence, there exists a $\mathbb{N}g\alpha^*$-open set $\mathbb{G}$ containing u but not v. Similarly, there exists a $\mathbb{N}g\alpha^*$-open set $\mathbb{H}$ containing v but not u. Thus, $\mathbb{U}$ is $\mathbb{N}g\alpha^*$-T_1.

Theorem 4.3: Let $f: (\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X})) \rightarrow (\mathbb{V}, \sigma_{\mathbb{R}}(\mathbb{Y}))$ be a bijection. If f is $\mathbb{N}g\alpha^*$-open and $\mathbb{U}$ is $\mathbb{N}T_1$, then $\mathbb{V}$ is $\mathbb{N}g\alpha^*$-T_1.

Proof: Suppose f is $\mathbb{N}g\alpha^*$-open and $\mathbb{U}$ is $\mathbb{N}T_1$. Let $v_1 \neq v_2 \in \mathbb{V}$. Since f is a bijection, there exist $u_1, u_2 \in \mathbb{U}$ such that $v_1 = f(u_1)$and $v_2 = f(u_2)$ with $u_1 \neq u_2$. Since $\mathbb{U}$ is $\mathbb{N}T_1$, there exist nano open sets $\mathbb{G}$ and $\mathbb{H}$ such that $u_1 \in \mathbb{G}$, $u_2 \notin \mathbb{G}$ and $u_2 \in \mathbb{H}, u_1 \notin \mathbb{H}$. Since f is $\mathbb{N}g\alpha^*$-open, $(\mathbb{G})$ and $f(\mathbb{H})$ are $\mathbb{N}g\alpha^*$-open sets in $\mathbb{V}$ such that $v_1 \in f(\mathbb{G})$, $v_2 \in f(\mathbb{H})$, $v_1 \notin f(\mathbb{H})$ and $v_2 \notin f(\mathbb{G})$. Thus $\mathbb{V}$ is $\mathbb{N}g\alpha^*$-T_1.

Theorem 4.4: Let $f: (\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X})) \rightarrow (\mathbb{V}, \sigma_{\mathbb{R}}(\mathbb{Y}))$ be a bijection. If f is $\mathbb{N}g\alpha^*$-closed and $\mathbb{U}$ is $\mathbb{N}T_1$, then $\mathbb{V}$ is $\mathbb{N}g\alpha^*$-T_1.

Proof: Suppose f is $\mathbb{N}g\alpha^*$-closed bijection and $\mathbb{U}$ is $\mathbb{N}T_1$. Let $v \in \mathbb{V}$. Since f is bijection, there exist $u \in \mathbb{U}$ such that $(u) = v$. Since $\mathbb{U}$ is $\mathbb{N}T_1$, by remark 2.15, $\{ u \}$ is closed in $\mathbb{U}$. Since f is $\mathbb{N}g\alpha^*$-closed, $(\{u\}) = \{v\}$. is $\mathbb{N}g\alpha^*$-closed. Since every singleton set in $\mathbb{V}$ is $\mathbb{N}g\alpha^*$- closed, by theorem 4.2 $\mathbb{V}$ is $\mathbb{N}g\alpha^*$-T_1.

Theorem 4.5: Let $f: (\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X})) \to (\mathbb{V}, \sigma_{\mathbb{R}}(\mathbb{Y}))$ be a bijection. If f is $\mathbb{N}g\alpha^*$-continuous and $\mathbb{V}$ is $\mathbb{N}T_1$, then $\mathbb{U}$ is $\mathbb{N}g\alpha^*$-T_1.

Proof: Suppose f is $\mathbb{N}g\alpha^*$-continuous and $\mathbb{V}$ is $\mathbb{N}T_1$. Let u_1, $u_2 \in \mathbb{U}$ with $u_1 \neq u_2$. Let $v_1 = f(u_1)$ and $v_2 = f(u_2)$. Since f is one to one , $v_1 \neq v_2$. Since $\mathbb{V}$ is $\mathbb{N}T_1$, there exist nano open sets $\mathbb{G}$ and $\mathbb{H}$ such that $v_1 \in \mathbb{G}$, $v_2 \notin \mathbb{G}$ and $v_1 \notin \mathbb{H}$, $v_2 \in \mathbb{H}$. Since f is a bijection, $u_1 \in f^{-1}(\mathbb{G})$, $u_2 \notin f^{-1}(\mathbb{G})$ and $u_2 \in f^{-1}(\mathbb{H})$,$u_1 \notin f^{-1}(\mathbb{H})$. Since f is $\mathbb{N}g\alpha^*$-continuous, $f^{-1}(\mathbb{G})$ and $f^{-1}(\mathbb{H})$ are $\mathbb{N}g\alpha^*$-open sets in $\mathbb{U}$. Thus $\mathbb{U}$ is $\mathbb{N}g\alpha^*$-T_1.

Theorem 4.6: Let $f: (\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X})) \to (\mathbb{V}, \sigma_{\mathbb{R}}(\mathbb{Y}))$ be a bijection. If f is $\mathbb{N}g\alpha^*$-irresolute and $\mathbb{V}$ is $\mathbb{N}g\alpha^*$- T_1, then $\mathbb{U}$ is $\mathbb{N}g\alpha^*$-T_1.

Proof: Suppose f is $\mathbb{N}g\alpha^*$-irresolute and $\mathbb{V}$ is $\mathbb{N}g\alpha^*$-T_1. Let u_1, $u_2 \in \mathbb{U}$, with $u_1 \neq u_2$. Let $v_1 = f(u_1)$ and $v_2 = f(u_2)$. Since f is one to one, $v_1 \neq v_2$. Since $\mathbb{V}$ is $\mathbb{N}g\alpha^*$-T_1, there exist $\mathbb{N}g\alpha^*$-open sets $\mathbb{G}$ and $\mathbb{H}$ such that $v_1 \in \mathbb{G}$, $v_2 \notin \mathbb{G}$ and $v_2 \in \mathbb{H}$, $v_1 \notin \mathbb{H}$. Since f is a bijection, $u_1 \in f^{-1}(\mathbb{G})$,$u_2 \notin f^{-1}(\mathbb{G})$ and $u_2 \in f^{-1}(\mathbb{H})$, $u_1 \notin f^{-1}(\mathbb{H})$. Since f is $\mathbb{N}g\alpha^*$-irresolute $f^{-1}(\mathbb{G})$ and $f^{-1}(\mathbb{H})$ are $\mathbb{N}g\alpha^*$-open sets in $\mathbb{U}$. Thus $\mathbb{U}$ is $\mathbb{N}g\alpha^*$-T_1.

V NANO GENERALIZED α^*-T_2 SPACES

Definition 5.1: A nano topological space $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$ is said to be nano generalized α^*-T_2 if whenever u and v are distinct points in $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$, there exist disjoint nano generalized α^*-open sets $\mathbb{G}$ and $\mathbb{H}$ in $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$ containing u and v respectively.

Theorem 5.2: For a nano topological space $\mathbb{U}$, the following statements are equivalent:

(i) $\mathbb{U}$ is $\mathbb{N}g\alpha^*$-T_2 space.

(ii) Let $u \in \mathbb{U}$. Then for each $u \neq v$, there exists a $\mathbb{N}g\alpha^*$-open $\mathbb{G}$ containing u such that $v \notin \mathbb{N}g\alpha^* cl(\mathbb{G})$.

(iii) For each $u \in \mathbb{U}$, $\cap\{ \mathbb{N}g\alpha^* cl(\mathbb{G}) : \mathbb{G}\ is\ \mathbb{N}g\alpha^*\text{-}open$ and $\in \mathbb{G} \} = \{u\}$.

Proof: (i)⇒(ii) Suppose $\mathbb{U}$ is a $\mathbb{N}g\alpha^*$-T_2. space. Let $u, v \in \mathbb{U}$ with $u \neq v$ Then there exist disjoint $\mathbb{N}g\alpha^*$-open sets $\mathbb{G}$ and $\mathbb{H}$ such that $u \in \mathbb{G}$ and $v \in \mathbb{H}$. Since $\mathbb{H}$ is $\mathbb{N}g\alpha^*$- open, $\mathbb{U} \setminus \mathbb{H}$ is $\mathbb{N}g\alpha^*$-closed and $\mathbb{G} \subseteq \mathbb{U} \setminus \mathbb{H}$. This implies that $\mathbb{N}g\alpha^*$ $(\mathbb{G}) \subseteq \mathbb{U} \setminus \mathbb{H}$. Since $v \notin \mathbb{U} \setminus \mathbb{H}$, $v \notin \mathbb{N}g\alpha^* cl(\mathbb{G})$.

(ii)⇒(iii) If $u \neq v$ then there exist a $\mathbb{N}g\alpha^*$-open set $\mathbb{G}$ such that $u \in \mathbb{G}$ and $v \notin \mathbb{N}g\alpha^* cl\ (\mathbb{G})$. Hence $v \notin \cap\{\mathbb{N}g\alpha^* cl\ (\mathbb{G}) : \mathbb{G}\ is\ \mathbb{N}g\alpha^*\text{-}open$ and $\in \mathbb{G}\ \}$. This proves (iii).

(iii)⇒(i) Let $u \neq v$ in $\mathbb{U}$. Then $v \notin \cap\{\ \mathbb{N}g\alpha^*\text{-}cl\ (\mathbb{G}) : \mathbb{G}\ is\ \mathbb{N}g\alpha^*\text{-}open$ and $\in \mathbb{G}\ \}$. This implies that there exist a $\mathbb{N}g\alpha^*$-open set $\mathbb{G}$ such that $u \in \mathbb{G}$ and $v \notin \mathbb{N}g\alpha^*\text{-}cl(\mathbb{G})$.Then $\mathbb{H} = \mathbb{U} \setminus \mathbb{N}g\alpha^*\text{-}cl\ (\mathbb{G})$ is $\mathbb{N}g\alpha^*$-open and $v \in \mathbb{H}$. Now, $\mathbb{G} \cap \mathbb{H} = \mathbb{G} \cap (\ \mathbb{U} \setminus \mathbb{N}g\alpha^*\text{-}\ cl\ (\mathbb{G}) \subseteq \mathbb{G} \cap (\mathbb{U} \setminus \mathbb{G}) = \phi$. This proves (i).

Theorem 5.3: Let $f : (\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X})) \to (\mathbb{V}, \sigma_{\mathbb{R}}(\mathbb{Y}))$ be a bijection. If f is $\mathbb{N}g\alpha^*$-open and $\mathbb{U}$ is $\mathbb{N}T_2$, then $\mathbb{V}$ is $\mathbb{N}g\alpha^*\text{-}\ T_2$.

Proof: Suppose f is $\mathbb{N}g\alpha^*$-open and $\mathbb{U}$ is $\mathbb{N}T_2$. Let $v_1 \neq v_2 \in \mathbb{V}$. Since f is a bijection, there exist $u_1, u_2 \in \mathbb{U}$ such that $v_1 = f\ (u_1)$ and $v_2 = f\ (u_2)$ with $u_1 \neq u_2$. Since $\mathbb{U}$ is $\mathbb{N}T_2$, there exist disjoint nano open sets $\mathbb{G}$ and $\mathbb{H}$ such that $u_1 \in \mathbb{G}$ and $u_2 \in \mathbb{H}$. Since f is $\mathbb{N}g\alpha^*$-open in $\mathbb{V}$ such that $v_1 \in (\mathbb{G})$, $v_2 \in f(\mathbb{H})$, $v_1 \notin f(\mathbb{H})$ and $v_2 \notin f(\mathbb{G})$. Thus $\mathbb{V}$ is $\mathbb{N}g\alpha^*\text{-}\ T_2$.

Theorem 5.4: Let $f : (\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X})) \to (\mathbb{V}, \sigma_{\mathbb{R}}(\mathbb{Y}))$ be a bijection. If f is $\mathbb{N}g\alpha^*$ -continuous and $\mathbb{V}$ is $\mathbb{N}T_2$, then $\mathbb{U}$ is $\mathbb{N}g\alpha^*\text{-}T_2$.

Proof: Suppose f is $\mathbb{N}g\alpha^*$-continuous and $\mathbb{V}$ is $\mathbb{N}T_2$. Let $u_1, u_2 \in \mathbb{U}$ with $u_1 \neq u_2$. Let $v_1 = f\ (u_1)$ and $v_2 = f\ (u_2)$. Since f is one to one , $v_1 \neq v_2$. Since $\mathbb{V}$ is $\mathbb{N}T_2$, there exist disjoint nano open sets $\mathbb{G}$ and $\mathbb{H}$ containing v_1 and v_2 respectively. Since f is $\mathbb{N}g\alpha^*$-continuous bijection, $f^{-1}(\mathbb{G})$ and $f^{-1}(\mathbb{H})$ are disjoint $\mathbb{N}g\alpha^*$-open sets in $\mathbb{U}$ containing u_1 and u_2 respectively. Thus $\mathbb{U}$ is $\mathbb{N}g\alpha^*\text{-}T_2$.

Theorem 5.5: Let $f : (\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X})) \to (\mathbb{V}, \sigma_{\mathbb{R}}(\mathbb{Y}))$ be a bijection. If f is a $\mathbb{N}g\alpha^*$-irresolute and $\mathbb{V}$ is $Ng\alpha^*\text{-}T_2$, then $\mathbb{U}$ is $\mathbb{N}g\alpha^*\text{-}T_2$.

Proof: Suppose f is $\mathbb{N}g\alpha^*$-irresolute and $\mathbb{V}$ is $\mathbb{N}g\alpha^*\text{-}T_2$. Let $u_1, u_2 \in \mathbb{U}$, with $u_1 \neq u_2$. Let $v_1 = f\ (u_1)$ and $v_2 = f\ (u_2)$. Since f is one to one, $v_1 \neq v_2$. Since $\mathbb{V}$ is $\mathbb{N}g\alpha^*\text{-}T_2$, there exist disjoint $\mathbb{N}g\alpha^*$-open sets $\mathbb{G}$ and $\mathbb{H}$ containing v_1 and v_2 respectively. Since f is $\mathbb{N}g\alpha^*$-irresolute bijection, $f^{-1}(\mathbb{G})$ and $f^{-1}(\mathbb{H})$ are disjoint $\mathbb{N}g\alpha^*$-open sets in $\mathbb{U}$ containing u_1 and u_2 respectively. Thus $\mathbb{U}$ is $\mathbb{N}g\alpha^*\text{-}T_2$.

VI CONCLUSION

In this paper we have introduced Nano generalized α^*- T_0 space and discussed some of its properties. Further more, Nano generalized α^*-T_1 space and Nano generalized α^*- T_2 space are defined and its properties are discussed.

VII REFERENCES

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