CONTR CONTINUOUS AND CONTRA IRRESOLUTE FUNCTIONS VIA NANO REGULAR*- OPEN SETS

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ABSTRACT: The purpose of this paper is to introduce and study a new class of functions called contra nano regular*- continuous and contra nano regular*- irresolute functions in nano topological spaces and investigate some of its properties.

KEY WORDS: Nano regular*- open, Nano regular* - continuous, Contra nano regular* - continuous

I INTRODUCTION

Lellis Thivagar et al. [4] introduced nano topological space with respect to a subset X of an universe which is defined in terms of lower and upper approximations of X. The elements of nano topological spaces are called the nano open sets. He also introduced the weak forms of nano open sets namely nano α-open sets, nano semi-open sets, nano regular-open sets and nano pre-open sets. In 2014, K.Bhuvaneswari and K.Mythili Gnanapriya[1] introduced Nano Generalised Closed Sets in Nano Topological Space. Quite recently the authors C. Reena, B.Santhalakshmi and S.M.Janu Priyadharshini[3] have introduced the concept of nano regular*-open sets and investigated its properties. M. Lellis Thivagar and Carmel Richard [2] introduced the concept of nano continuity in nano topological spaces. In this paper, I have introduced a new concept contra nano regular* - continuous functions and contra nano regular*- irresolute functions and its properties in nano topological spaces.

II PRELIMINARIES

Throughout this paper (U,τ_R(X)), (V,σ_R(Y)), (W,η_R(Z)) represent non empty nano topological spaces on which no separation axioms are assumed unless otherwise mentioned.

Definition 2.1[1]: Let (U, τ_R(X)) be a Nano topological space. A subset A of (U, τ_R(X)) is called ***Nano generalized-closed set*** if Ncl(A) ⊆ V where A ⊆ V and V is Nano-open.

The complement of Nano generalized -closed set is called as ***Nano generalized-open set***.

Definition 2.2[1]: For every set A ⊆ U,

(i) ***Nano generalized closure of A*** is defined as the intersection of all Ng- closed sets containing A and is denoted by Ncl*(A).

(ii) ***Nano generalized interior of A*** is defined as the union of all Ng- open sets contained in A and is denoted by Nint*(A).

Definition 2.3[3]: A subset A of a nano topological space $(U, \tau_R(X))$ is called **nano regular*- open** if $A = NInt(NCl^*(A))$. The collection of all nano regular*-open sets is denoted by **NR*O**$\boldsymbol{(U, \tau_R(X))}$.

Definition 2.4[3]: The complement of nano regular* -open set is called as **nano regular*-closed**. The collection of all nano regular* - closed sets is denoted by **NR*C**$\boldsymbol{(U, \tau_R(X))}$.

Theorem 2.5: [4] (i) Every nano regular- closed set is nano regular*-closed

(ii) Every nano regular*-closed set is nano closed

(iii) Every nano regular*- closed set is nano semi closed

(iv) Every nano regular*- closed set is nano pre closed.

(v) Every nano regular*-closed set is nano α-closed

(vi) Every nano regular*-closed set is nano b-closed

(vii)Every nano regular*-closed set is nano semi*-closed.

Theorem 2.6: [2] (i) Every nano regular*-closed set is nano β-closed

(ii) Every nano regular*-closed set is nano β*-closed.

Definition 2.7: A function f: (U,τ_R(X)) → (V,σ_R(Y)) is said to be

(i) contra nano continuous if $f^{-1}(V)$ is nano closed in (U,τ_R(X)) for every nano open set V in (Y,σ_R(Y)).

(ii) contra nano regular-continuous if $f^{-1}(V)$ is nano regular-closed in (U,τ_R(X)) for every nano open set V in (Y,σ_R(Y)).

(iii) contra nano α-continuous if $f^{-1}(V)$ is nano α-closed in (U,τ_R(X)) for every nano open set V in (Y,σ_R(Y)).

(iv) contra nano semi*-continuous if $f^{-1}(V)$ is nano semi*-closed in (U,τ_R(X)) for every nano open set V in (Y,σ_R(Y)).

III CONTRA NANO REGULAR* - CONTINUOUS FUNCTIONS

Definition 3.1: Let (U,τ_R(X)) and (V,σ_R(Y)) be two nano topological spaces.A function f: (U, τ_R(X)) → (V, σ_R(Y)) is said to be **contra nano regular* continuous** if f^{-1}(V) is nano regular* closed set in (U,τ_R(X)) for every nano open set V in (V,σ_R(Y)).

Example 3.2: Let $U=\{a,b,c\}$,U/R=$\{\{a\},\{b\}\}$ and X=$\{a,c\}$.Then $\tau_R(X)=\{U,\emptyset,\{a\}\}$ and $NR^{*}O(U,\tau_R(X))$=$\{U,\emptyset,\{a\}\}$ $NR^{*}C(U,\tau_R(X))$=$\{\{U,\emptyset,\{b,c\}\}$. Let V=$\{a,b,c\}$,V/R=$\{\{a\},\{c\}\}$and Y=$\{a,b,c\}$.Then $\sigma_R(Y)$=$\{V,\emptyset,$\{a,c\}$\}$.Define f:(U,τ_R(X)) → (V,σ_R(Y)) by $f(a)=b$,$f(b)=a$,$f(c)=c$.since $f^{-1}(\{a,c\})=\{b,c\}$ is nano regular⋆ closed in (U,τ_R(X)),Hence f is contra Nano regular⋆ continuous.

Theorem 3.3: Every contra nano regular- continuous function is contra nano regular*-continuous.

Proof: Let f: (U,τ_R(X)) → (V,σ_R(Y)) be contra nano regular-continuous. Let V be any nano open set in (V,σ_R(Y)). Then $f^{-1}(V)$ is nano regular closed in (U,τ_R(X)). Hence by theorem 2.5(i), $f^{-1}(V)$ is nano regular*-closed in (U,τ_R(X)). Therefore f is contra nano regular*-continuous.

Remark 3.4 :

The converse of the above theorem is not true as shown in the following example.

Example 3.5: Let U = {a,b,c}, X = {a,c} and U/R = {{a},{b}}. Then τ_R(X) = {U,∅,{a}}, NR*O(U,$\tau_R(X)$)= {U,∅,{a}}, NR*C(U,$\tau_R(X)$={U,∅{b,c}} and NRO(U,$\tau_R(X)$)= {U,∅}, NRC(U,$\tau_R(X)$)= {U,∅}. Let V=$\{a,b,c\}$,V/R=$\{\{a\},\{c\}\}$and Y=$\{a,b,c\}$.Then $\sigma_R(Y)$=$\{V,\emptyset,$\{a,c\}$\}$.Define f: (U,τ_R(X)) → (V,σ_R(Y)) by f(a) = b; f(b) = a; f(c) = c. Then f is contra nano regular*-continuous but not nano contra regular-continuous. Since for the nano open set {b,c} in (V,σ_R(Y)), f^{-1}(a,c) = {b,c} is nano regular*-closed but not nano regular closed in (U,τ_R(X)).

Theorem 3.6: Every contra nano regular*-continuous function is contra nano continuous.

Proof: Let f: (U,τ_R(X)) → (V,σ_R(Y)) be contra nano regular*-continuous. Let V be any nano open set in (V,σ_R(Y)). Then $f^{-1}(V)$ is nano regular*-closed in (U,τ_R(X)). Hence by theorem 2.5(ii), $f^{-1}(V)$ is nano-closed in (U,τ_R(X)). Therefore, f is contra nano-continuous.

Remark 3.7: The converse of the above theorem is not true as shown in the following example.

Example 3.8: Let U ={a,b,c}, X = {b,c} and U/R = {{a},{b},{c}}. Then τ_R(X) = {U,∅,{b,c} and NR*O((U,τ_R(X)) = {U,∅},NR*C((U, τ_R(X)) ={U,∅} . Let V = {a,b,c}, Y = {a,b} and V/R = {{a},{b,c}}. Then σ_R(Y) = {V,∅,{a},{b,c}}. Define f: (U,τ_R(X)) → (V,σ_R(Y)) by f(a) = a; f(b) = b; f(c) = c. Then f is contra nano-continuous but not contra nano regular*-continuous. Since for the nano open set {a} in (V,σ_R(Y), f^{-1}(a) =

{a} and {b,c} in (V,σ_R(Y), f^{-1}(b,c)={b,c} is nano-closed but not nano regular*-closed in (U,τ_R(X)).

Theorem 3.9: Every contra nano regular*-continuous function is contra nano semi-continuous.

Proof: Let f: (U,τ_R(X)) → (V,σ_R(Y)) be contra nano regular*-continuous. Let V be any nano open set in (V,σ_R(Y)). Then $f^{-1}(V)$ is nano regular*-closed in (U,τ_R(X)). Hence by theorem 2.5(iii), $f^{-1}(V)$ is nano semi-closed in (U,τ_R(X)). Therefore, f is contra nano semi-continuous.

Remark 3.10: The converse of the above theorem is not true as shown in the following example.

Example 3.11: Let U ={a,b,c}, X = {a,c} and U/R = {{a},{b}}. Then τ_R(X) = {U,∅,{a}}, NR*O(U,τ_R(X)) = {U,∅,{a}}, NR*C((U, τ_R(X)) ={U,∅,$\{b,c\}$} and NSO(U,τ_R(X)) = {U,∅,{a},{a,b},{a,c}}, NSC(U,τ_R(X)) = {U,∅,{b,c}{c},{b}}. Let V ={a,b,c},V/R={{a},{b}},Y={a,c}. Then σ_R(Y) = {V,∅,{a,}}. Define f: (U,τ_R(X))→ (V,σ_R(Y)) by f(a) = c; f(b) = a; f(c) = b. Then f is contra nano semi-continuous but not contra nano regular*-continuous. Since for the nano open set {a} in (V,σ_R(Y)), f^{-1}(a) = {b} is nano semi-closed but not nano regular*-closed in (U,τ_R(X)).

Theorem 3.12: Every contra nano regular*-continuous function is contra nano pre-continuous

Proof: Let f: (U,τ_R(X)) → (V,σ_R(Y)) be contra nano regular*-continuous. Let V be any nano open set in (V,σ_R(Y)). Then $f^{-1}(V)$ is nano regular*-closed in (U,τ_R(X)). Hence by theorem 2.5(iv), $f^{-1}(V)$ is nano pre-closed in (U,τ_R(X)). Therefore, f is contra nano pre-continuous.

Remark 3.13: The converse of the above theorem is not true as shown in the following example.

Example 3.14: Let U = {a,b,c}, X = {b,c} and U/R = {{a},{b},{c}}. Then τ_R(X) = {U,∅,{b,c}}, NR*O(U,τ_R(X)) ={U,∅}, NR*C(U, τ_R(X))={U,∅} and NPO(U,τ_R(X)) ={U,∅,{b},{c},{a,b},{a,c},{b,c}}, NPC(U, τ_R(X))= {U,∅,{a,c},{a,b},{c},{b},{a}}. Let V ={a,b,c}, Y = {a,b} and V/R = {{a},{b,c}}. Then σ_R(Y) = {V,∅,{a},{b,c}}. Define f: (U,τ_R(X)) → (V,σ_R(Y)) by f(a) = b; f(b) = a; f(c) = c. Then f is contra nano pre-continuous but not contra nano regular*-continuous. Since for the nano open set {a} in (V,σ_R(Y)), f^{-1}(a) = {b} and {b,c} in (V,σ_R(Y)), f^{-1}(b,c) = {a,c} is contra nano pre-closed but not contra nano regular*-closed in (U,τ_R(X)).

Theorem 3.15: Every contra nano regular*-continuous function is contra nano α-continuous.

Proof: Let f: (U,τ_R(X)) → (V,σ_R(Y)) be contra nano regular*-continuous. Let V be any nano open set in (V,σ_R(Y)). Then $f^{-1}(V)$ is nano regular*-closed in (U,τ_R(X)). Hence by theorem 2.5(v), $f^{-1}(V)$ is nano α- closed in (U,τ_R(X)). Therefore, f is contra nano α-continuous.

Remark 3.16: The converse of the above theorem is not true as shown in the following example.

Example 3.17: Let U = {a,b,c}, X ={b,c} and U/R = {{a},{b},{c}}. Then τ_R(X) = {U,∅,{b,c}}, NR*O(U,τ_R(X)) = { U,∅},NR*C(U, τ_R(X))={U,∅ }and NαO(U,τ_R(X)) = {U,∅,{b,c},NαC(U, τ_R(X))={U,∅,{a}}. Let V = {a,b,c}, V/R={{a},{b}},Y={a,c}. Then σ_R(Y) = {V,∅,{a}}.Define f: (U,τ_R(X)) → (V,σ_R(Y)) by f(a) = a; f(b) = b; f(c) = c. Then f is contra nano α-continuous but not contra nano regular* -continuous. Since for the nano open set {a} in (V,σ_R(Y)), f^{-1}(a) = {a} is nano α- closed but not nano regular*- closed in (U,τ_R(X)).

Theorem 3.18: Every contra nano regular*-continuous function is contra nano β-continuous.

Proof: Let f: (U,τ_R(X)) → (V,σ_R(Y)) be contra nano regular*-continuous. Let V be any nano open set in (V,σ_R(Y)). Then $f^{-1}(V)$ is nano regular*-closed in (U,τ_R(X)). Hence by theorem 2.6(i), $f^{-1}(V)$ is nano β- closed in (U,τ_R(X)). Therefore, f is contra nano β-continuous.

Remark 3.19: The converse of the above theorem is not true as shown in the following example.

Example 3.20: Let U = {a,b,c}, X = {a,c} and U/R = {{a},{b}}. Then τ_R(X) = {U,∅,{a}},NβO(U,τ_R(X)) ={U,∅,{a},{a,b},{a,c}},NβC(U, τ_R(X))={U,∅,{b,c},{c}, {b}} and NR*O(U,τ_R(X)) ={U,∅,{a}},NR*C(U, τ_R(X))={U,∅,{b,c}}. Let V = {a,b,c}, Y = {a,b,c} and V/R={{a},{b}}. Then σ_R(Y) = {V,∅,{a}}. Define f: (U,τ_R(X)) → (V,σ_R(Y)) by f(a) = b; f(b) = a; f(c) = c. Then f is contra nano β-continuous but not contra nano regular*-continuous. Since for the nano open set {a} in (V,σ_R(Y)), f^{-1}(a) = {b} is nano β- closed but not nano regular*-closed in (U,τ_R(X)).

Theorem 3.21: Every contra nano regular*-continuous funcion is contra nano b-continuous.

Proof: Let f: (U,τ_R(X)) → (V,σ_R(Y)) be contra nano regular*-continuous. Let V be any nano open set in (V,σ_R(Y)). Then $f^{-1}(V)$ is nano regular*-closed in (U,τ_R(X)). By theorem 2.5(vi), $f^{-1}(V)$ is nano b-closed in (U,τ_R(X)). Therefore, f is contra nano b-continuous.

Remark 3.22: The converse of the above theorem is not true as shown in the following example.

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Example 3.23: Let U = {a,b,c,d}, X = {a,b,c} and U/R = {{b},{d},{c}}. Then τ_R(X) = {U,∅,{b,c}},NbO(U,τ_R(X)) ={U,∅,{b},{c},{a,b}{a,c},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}},NbC(U, τ_R(X))={U,∅,{a,c,d},{a,b,d},{c,d}{b,d},{a,d},{a,c},{a,b},{d},{c},{b},{a}}and NR*O(U,τ_R(X)) = {U,∅,{b,c}},NR*C(U, τ_R(X)) = {U,∅,{a,d}}. Let V = {a,b,c,d}, Y = {a,c,d} and V/R = {{a},{b}}. Then σ_R(Y) = {V,∅,{a}}. Define f: (U,τ_R(X)) → (V,σ_R(Y)) by f(a) = b; f(b) = a; f(c) = c; f(d) = d. Then f is contra nano b-continuous but not contra nano regular*-continuous. Since for the nano open set {a} in (V,σ_R(Y), f^{-1}(a) = {b} is nano b- closed but not nano regular*- closed in (U,τ_R(X)).

Theorem 3.24: Every contra nano regular*-continuous function is contra nano β*-continuous.

Proof: Let f: (U,τ_R(X)) → (V,σ_R(Y)) be contra nano regular*-continuous. Let V be any nano open set in(V,σ_R(Y)). Then f^{-1}(V) is nano regular*-closed in (U,τ_R(X)). By theorem 2.6(ii), f^{-1}(V) is nano β*-closed in (U,τ_R(X)). Therefore f is contra nano β*-continuous.

Remark 3.25: The converse of the above theorem is not true as shown in the following example.

Example 3.26: Let U = {a,b,c,d}, X = {a,c,d} and U/R = {{a,b},{c,d}}. Then τ_R(X) = {U,∅.{c,d},{a,b}},NR*O(U,τ_R(X))={U,∅,{a,b},{c,d}},NR*C(U, τ_R(X))={U,∅,{{c,d},{a,b}}and

Nβ*O(U,τ_R(X))={U,∅,{a},{b},{c},{d},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}},Nβ*C(U, τ_R(X))={U,∅,{b,c,d},{a,c,d},{a,b,d},{a,b,c},{c,d},{b,d},{b,c},{a,d},{a,c},{a,b}{d},{c},{b},{a}} . Let V = {a,b,c,d}, Y = {a,c,d} and V/R = {{a},{b}}. Then σ_R(Y) = {V,∅,{a}}. Define f: (U,τ_R(X)) → (V,σ_R(Y)) by f(a) =a; f(b) =b; f(c) =c. Then f is contra nano β*-continuous but not contra nano regular*-continuous. Since for the nano open set {a} in (V,σ_R(Y)), f^{-1}(a) ={a} is nano β*- closed but not nano regular*-closed in (U,τ_R(X)).

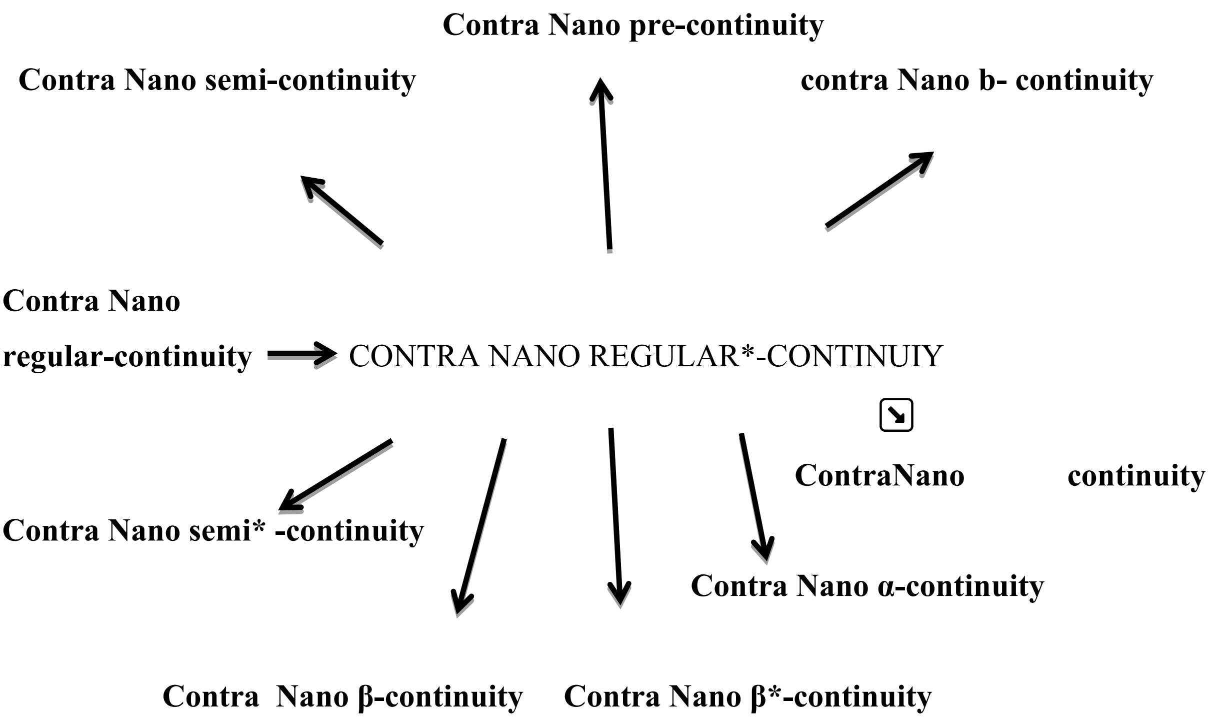
Theorem 3.27: Every contra nano regular*-continuous function is contra nano semi*-continuous.

Proof: Let f: (U,τ_R(X)) → (V,σ_R(Y)) be contra nano regular*-continuous. Let V be any nano open set in (V,σ_R(Y)). Then f^{-1}(V) is nano regular*- closed in (U,τ_R(X)). By theorem 2.5(vii), f^{-1}(V) is nano semi*- closed in (U,τ_R(X)). Therefore f is contra nano semi*-continuous.

Remark 3.28: The converse of the above theorem is not true as shown in the following example.

Example 3.29: Let U = {a,b,c}, X = {b,c} and U/R = {{a},{b},{c}}. Then τ_R(X) = {U,∅.{b,c}}, NR*O(U,τ_R(X)) = {U,∅},NR*C(U, τ_R(X))={U,∅} and NS*O(U,τ_R(X)) ={U,∅,{b,c}}, NS*C(U, τ_R(X))={U,∅,{a}}. Let V= {a,b,c}, V/R={{a},{b}},Y={a,c}. Then σ_R(Y) = {V,∅,{a}}. Define f: (U,τ_R(X)) → (V,σ_R(Y)) by f(a) = a; f(b) = b; f(c) = c. Then f is contra nano semi*-continuous but not contra nano regular*-continuous. Since for the nano open set {a} in (V,σ_R(Y)),f^{-1}(a) = {a} is nano semi*- closed but not nano regular*-closed in (U,τ_R(X)).

From the above discussions, we have the following diagram:



Theorem 3.30: A function f: (U,τ_R(X)) → (V,σ_R(Y)) is contra nano regular*-continuous if and only if f^{-1}(F) is nano regular*- open in (U,τ_R(X)) for every closed set F in (V,σ_R(Y)).

Proof: Let f: (U,τ_R(X)) → (V,σ_R(Y)) be contra nano regular*-continuous and F be a closed set in (V,σ_R(Y)). Then G =V\F is nano open in (V,σ_R(Y)). Then f^{-1}(G) is nano regular*-closed in (U,τ_R(X)). Therefore f^{-1}(F) = $f^{-1}(V\backslash G)$ = U\\$f^{-1}(G)$ is nano regular*- open in (U,τ_R(X)).

Converse: Let G be any nano open set in (V,σ_R(Y)). Then F =$(V\backslash G)$ is nano closed in (V,σ_R(Y)). By assumption f^{-1}(F) is nano regular*-closed in (U,τ_R(X)). Hence f^{-1}(G)

= $f^{-1}(V\backslash F)$ = U\\$f^{-1}(F)$ is nano regular*-open in (U,τ_R(X)). Therefore, f is contra nano regular*- continuous.

Theorem 3.31: If a function f: (U, τ_R(X)) → (V, σ_R(Y)) is nano regular*-continuous and if (V, σ_R(Y)) is locally indiscrete, then f is contra nano regular*-continuous.

Proof: Let G be an nano open set (V, σ_R(Y)). Since (V, σ_R(Y)) is locally discrete, G is nano closed in (V, σ_R(Y)). Since f is nano regular*-continuous, $f^{-1}(G)$ is nano regular*-closed in (U, τ_R(X)) .Therefore f is contra nano regular*-continuous.

Theorem 3.32: The composition of two contra nano regular*-continuous functions is nano regular*-continuous.

Proof: Let f: (U,τ_R(X)) → (V,σ_R(Y)) and g: (V, σ_R(Y)) → (W, η_R(Z)) be contra nano regular*-continuous functions. Let V be a nano open set in (W, η_R(Z)).Since g is contra nano regular*-continuous, $g^{-1}(V)$ nano regular*-closed in (V,σ_R(Y)). Since theorem 2.5(ii), g^{-1}(V) is nano closed in (V,σ_R(Y)). Since f is contra nano regular*-continuous $f^{-1}(g^{-1}(V))$ is nano regular*- open in (U,τ_R(X)). Thus g ∘ f is nano regular*-continuous.

Theorem 3.33: Let f: (U, τ_R(X)) → (V, σ_R(Y)) be nano regular*-continuous and g:(V, σ_R(Y) → (W, η_R(Z)) be contra nano regular*-continous function. Then ($g \circ f$):(U, τ_R(X)) → (W,η_R(Z)) is contra nano regular*-continuous.

Proof: Let S be nano closed set in (W,η_R(Z)) . Since g is contra nano regular*-continuous, $g^{-1}(S)$ is nano regular*-open in (V, σ_R(Y)). By theorem 2.5(ii) $g^{-1}(S)$ is nano open in (V,σ_R(Y)). Also $f$is nano regular* -continuous, $f^{-1}(g^{-1}(S))$=$(g \circ f)^{-1}(S)$ is nano regular*-open in(U, τ_R(X)). Hence by theorem 3.30, $g \circ f$ is contra nano regular*-continuous.

Theorem 3.34: Let f: (U, τ_R(X)) → (V, σ_R(Y)) be nano regular* -irresolute and g:(V, σ_R(Y) → (W, η_R(Z)) be contra nano regular*-continous function. Then $g \circ f$:(U, τ_R(X)) → (W,η_R(Z)) is contra nano regular*-continuous.

Proof: Let G be nano open set in (W,η_R(Z)) . Since g is contra nano regular* -continuous, $g^{-1}(G)$ is nano regular*-closed in (V, σ_R(Y)). Also, since f is nano regular*-irresolute, $f^{-1}(g^{-1}(G))$=$(g \circ f)^{-1}(G)$ is nano regular*-closed in (U, τ_R(X)). Hence $g \circ f$ is contra nano regular* -continuous.

IV CONTRA NANO REGULAR*-IRRESOLUTE FUNCTIONS

Definition 4.1: Let $(\mathrm{U}, \tau_R(\mathrm{X}))$ and $(\mathrm{V}, \sigma_R(\mathrm{Y})$ be two nano topological spaces. A function $f: (\mathrm{U}, \tau_R(\mathrm{X})) \to (\mathrm{V}, \sigma_R(\mathrm{Y}))$ is said to be **contra nano** regular***-irresolute** if $f^{-1}(\mathrm{V})$ is nano regular*-closed in $(\mathrm{U}, \tau_R(\mathrm{X}))$ for every nano regular*-open set V in $(\mathrm{V}, \sigma_R(\mathrm{Y})$.

Example 4.2: Let U = {a,b,c,d}, X = {a,c,d} and U/R = {{a,b},{c,d}}. Then τ_R(X) = {U,∅,{c,d},{a,b}} and NR*O(U,τ_R(X)) = {U,∅,{a,b},{c,d}},NR*C(U, τ_R(X)) = {U,∅,{c,d},{a,b}}. Let V = {a,b,c,d}, Y = {a,b,c} and V/R = {{b},{d},{c}}. Then σ_R(Y)= {V,∅,{b,c}} and NR*O(V,σ_R(Y)) = {V,∅,{b,c}},NR*C(V, σ_R(Y)) = {V,∅,{a,d}}. Define f: (U,τ_R(X)) → (V,σ_R(Y)) by f(a) = a; f(b) =d; f(c) =b; f(d) =c. Then f^{-1}{b,c} = {c,d} is nano regular*- closed in (U,τ_R(X)). Hence f is contra nano regular*-irresolute function.

Theorem 4.3: A function $f: (\mathrm{U}, \tau_R(\mathrm{X})) \to (\mathrm{V}, \sigma_R(\mathrm{Y}))$ is contra nano regular*-irresolute if and only if the inverse image of every nano regular*-closed set in $(\mathrm{V}, \sigma_R(\mathrm{Y}))$ is nano regular*-open in $(\mathrm{U}, \tau_R(\mathrm{X}))$.

Proof: Let $f: (\mathrm{U}, \tau_R(\mathrm{X})) \to (\mathrm{V}, \sigma_R(\mathrm{Y}))$ be a contra nano regular* -irresolute and S be a nano regular* - closed set in $(\mathrm{V}, \sigma_R(\mathrm{Y}))$. Since f is contra nano regular* -irresolute function, $f^{-1}(S^c) = (f^{-1}(S))^c$ is nano regular* -closed in (U,τ_R(X)).Hence $f^{-1}(S)$ is nano regular* -open set in (U,τ_R(X)).Conversely, suppose the inverse image of every nano regular* -closed set in (V,σ_R(Y)) is nano regular*-open in (U,τ_R(X)) .Let S be a nano regular* -open set in (V,σ_R(Y)) . Then S^c is nano regular*-closed in (V,σ_R(Y)) .By assumption, $f^{-1}(S^c) = (f^{-1}(S))^c$ is nano regular* -open in (U,τ_R(X)) .Hence $f$is contra nano regular*-irresolute.

Theorem 4.4: Let $f: (\mathrm{U}, \tau_R(\mathrm{X})) \to (\mathrm{V}, \sigma_R(\mathrm{Y}))$ be nano regular*-irresolute and$g$:$(V, \sigma_R(Y) \to (W, \eta_R(Z)$ be contra nano regular*-irresolute.Then $(g \circ f): (\mathrm{U}, \tau_R(\mathrm{X})) \to (W, \eta_R(Z))$ is contra nano regular*-irresolute.

Proof: Let G be a nano regular*-open set in $(W, \eta_R(Z))$.Since g is contra nano regular*-iresolute, $g^{-1}(G)$ is nano regular*-closed in $(\mathrm{V}, \sigma_R(\mathrm{Y}))$ .Since $f$is nano regular*-irresolute,$(g \circ f)^{-1}(G) = f^{-1}(g^{-1}(G))$ is nano regular*-closed in $(\mathrm{U}, \tau_R(\mathrm{X}))$.Hence $g \circ f$ is contra nano regular*-irresolute.

Theorem 4.5: Let $f: (\mathrm{U}, \tau_R(\mathrm{X})) \to (\mathrm{V}, \sigma_R(\mathrm{Y}))$ be contra nano regular*-irresolute and g:$(V, \sigma_R(Y) \to (W, \eta_R(Z)$ be nano regular*-irresolute. Then $(g \circ f): (\mathrm{U}, \tau_R(\mathrm{X})) \to (W, \eta_R(Z)$ is contra nano regular*-irresolute.

Proof: Let G be a nano regular*-open set in $(W, \eta_R(Z))$.Since g is nano regular*-irresolute, $g^{-1}(G)$ is nano regular*-open in $(\mathrm{V}, \sigma_R(\mathrm{Y}))$ .Since $f$is contra nano regular*-irresolute,$(g \circ f)^{-1}(G) = f^{-1}(g^{-1}(G))$ is nano regular*-closed in $(\mathrm{U}, \tau_R(\mathrm{X}))$.Hence $g \circ f$ is contra nano regular*-irresolute.

Theorem 4.6: If $f: (\mathrm{U}, \tau_R(\mathrm{X})) \rightarrow (\mathrm{V}, \sigma_R(\mathrm{Y}))$ and $g:(V, \sigma_R(Y) \rightarrow (W, \eta_R(Z)$ are contra nano regular*-irresolute functions, then $g \circ f: (\mathrm{U}, \tau_R(\mathrm{X})) \rightarrow (W, \eta_R(Z)\ )$ is nano regular*-irresolute.

Proof: Let G be nano regular*-open set in Z. Since g is contra nano regular*-irresolute, $g^{-1}(G)$ is nano regular*-closed in $(\mathrm{V}, \sigma_R(\mathrm{Y}))$. Also, since f is contra nano regular*-irresolute, by theorem 4.3, $(g \circ f)^{-1}(G) = f^{-1}(g^{-1}(G))$ is nano regular*-open in $(\mathrm{U}, \tau_R(\mathrm{X}))$. Hence $g \circ f$ is nano regular*-irresolute.

V CONCLUSION

In this paper we have introduced contra nano regular*-continuous and contra nano regular*-irresolute functions and investigated their fundamental properties. Also, we have examined their relationships with other existing nano continuous and nano irresolute functions.

VI REFERENCES

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