Contra $\mathfrak{M}\alpha^*$-Continuous and Contra $\mathfrak{M}\alpha^*$-Irresolute Functions in Micro Topoigical Spaces

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Abstract: The aim of this paper is to introduce and investigate the notion of contra $\mathfrak{M}\alpha^*$-continuous and contra $\mathfrak{M}\alpha^*$-irresolute functions.We also examine some of their properties and relations of other such functions.

Keywords: Micro α^*- open maps, Micro α^* - closed maps, Micro α^*- open sets, Micro α^*-closed sets, Micro α - continuous, Micro α^*- continuous, Micro α - irresolute, Micro α^* - irresolute.

I.INTRODUCTION

The concept of Nano topology was introduced by Thivagar et al which is defined in terms of the lower and upper approximation and the boundary region of a subset of an universe. The notion of approximations and boundary region of a set was originally proposed by Pawlak [7] in order to introduce the concept of rough set theory. In 2019, [3] Chandraseker introduced the concept of micro topology which is a simple extension of nano topology and so studied the concepts of Micro pre-open and Micro semi-open sets.[4] K. Chandraseker introduced the concept of micro topology which is a simple extension of Nano topology and so studied the concepts of Micro pre - open and Micro semi - open sets. [4] K. Chandrasekar and swathi introduced Micro α –open in micro topological space.P. Anbarasi Rodrigo[1] introduced $\mathfrak{M}\alpha^*$ open sets and studied their properties. Dontchev [5] introduced the notions of contra continuity in topological spaces.

II.PRELIMINARIES

Definition 2.1[3]

Let U be a nonempty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space.

Let $X \subseteq U$

1. The **lower approximation of X** with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is $L_R(X) = U_{(x\in U)}\{R(x): R(x) \subseteq X\}$ where $R(x)$denotes the equivalence class determined by $x \in U$.

2. The **upper approximation of X** with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$.That is $U(X) = U_{(x\in U)}\{R(x): R(x) \cap X \neq \emptyset\}\ x \in U$.

3. The **boundary region of X** with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. That is $B_R(X) = U_R(X) - L_R(X)$.

Definition: **2.2[3]**

Let U be an universe, R be an equivalence relation on U and $\tau R(X) = \{U, \emptyset, LR(X), R(X), BR(X)\}$ where $X \subseteq U$ satisfies the following axioms.

1. $U, \emptyset \in \tau R(X)$

2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

3. The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.Then $\tau_R(X)$ is called the **Nano topology** on U with respect to X. The space $(U, \tau_R(X))$ is the Nano topological space. The elements are called Nano open sets.

Definition: 2.3[3]

$(U, \tau_R(X))$ is a nano topological space here $\mu_R(X) = \{N \cup (N' \cap \mu)\} / N, N' \in \tau_R(X)$ and it is called **Micro topology** of $\tau_R(X)$ by μ where $\mu \notin \tau_R(X)$. The micro topology $\mu_R(X)$ satisfies the following axioms.

1. $U, \emptyset \in \mu_R(X)$

2. The union of the elements of any subcollection of $\mu_R(X)$ is in $\mu_R(X)$.

3. The intersection of the elements of any finite sub collection of $\mu_R(X)$ is in $\mu_R(X)$.

Then $\mu_R(X)$ is called the micro topology on U with respect to X. The triplet $(U, \tau_R(X), \mu_R(X))$ is called micro topological spaces and the elements of $\mu_R(X)$ are called micro open sets and the complement of a micro open set is called a micro closed set.

Example: 1

Let $U = \{p, q, r, s, \}$ and $U / R = \{\{p\}, \{q, r, s\}, \{t\}\}$ and let $X = \{q, r\} \subseteq U$. Then $N\tau_R(X) = \{U, \emptyset, \{q, r, s\}\}$ and $\mu = \{p\}$. $\mathfrak{M}$ - open, $\mu_R(X) = \{u, \emptyset, \{p\}, \{p, q, r, s\}, \{q, r, s\}\}$.

Definition: 2.4[3]

The **micro closure** of a set A is denoted by $\mathfrak{M}$ - cl(A) and is defined as $\mathfrak{M}$ - cl(A) $= \cap \{B: B$ *is* micro closed *and* $A \subseteq B\}$.

Definition: 2.5[3]

The **micro interior** of a set A is denoted by $\mathfrak{M}$ - int(A) and is defined as $\mathfrak{M}$ - int(A) $= \cup \{B: B$ *is* micro open *and* $A \supseteq B\}$.

Definition: **2.6[4]**

Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space. A set A is called an $\mathfrak{M}\alpha$ - open set if $A \subseteq \mathfrak{M}$ -int ($\mathfrak{M}$ - cl($\mathfrak{M}$ - int(A))), The complement of a $\mathfrak{M}\alpha$ - open set is called an **$\mathfrak{M}\alpha$ - closed set.**

Definition : 2.7[4]

Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau_R{}'(Y), \mu_R{}'(Y))$ be two Micro topological spaces. A function $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R{}'(Y), \mu_R{}'(Y))$ is called **Micro - continuous function** if $f^{-1}(H)$ is Micro open in U for every Micro - open H in V.

Definition : 2.8[2]

Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau_R{}'(Y), \mu_R{}'(Y))$ be two Micro topological spaces.A function $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R{}'(Y), \mu_R{}'(Y))$ is called **Micro α- continuous function** if $f^{-1}(H)$ is Micro α open in U for every Micro - open H in V.

Definition : 2.9[2]

Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau_R{}'(Y), \mu_R{}'(Y))$ be two Micro topological spaces.A function $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R{}'(Y), \mu_R{}'(Y))$ is called **Micro α^*- continuous function** if $f^{-1}(H)$ is Micro α^* open in U for every Micro - open H in V.

III.Contra $\mathfrak{M}\alpha^*$ – Continuous Functions.

Definition : 3.1

A function $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R{}'(X), \mu_R{}'(Y))$ is contra $\mathfrak{M}$- continuous if $f^{-1}(O)$ is $\mathfrak{M}$- closed in $(U, \tau_R(X), \mu_R(X))$ for every $\mathfrak{M}$ - open set O in $(V, \tau_R{}'(Y), \mu_R{}'(Y))$.

Definition: 3.2

A function $f:\left(U,\tau_R(X),\mu_R(X)\right)\to(V,\tau_R{}'(Y),\mu_R{}'(Y))$ is contra $\mathfrak{M}\alpha^*$ - continuous if $f^{-1}(O)$ is $\mathfrak{M}\alpha^*$- closed in $(U,\tau_R\ (X),\mu_R(X))$ for every $\mathfrak{M}$ - open set in $(V,\tau_R{}'(Y),\mu_R{}'(Y))$.

Example : 3.3

Let $U=V=\{a,b,c\}$ with $U/R=\{\{a,b\},\{b\}\}$ and $X=\{a,c\}\subseteq U and\ \mu=\{b\}\notin\tau_R(X)$.Then $\tau_R(X)$=$\{U,\emptyset,\{a,c\}\}$and $\mu_R(X)=\{U,\emptyset,\{a,c\},\{b\}\}$ then $\mu_R{}^c(X)=\{U,\emptyset,\{b\},\{a,c\}$.Let $Y=\{c\}\subseteq V$ with $V/R=\{\{c\},\{a,b\}\}$. Then $\mu=\{a\}\notin\tau_R(Y)$ and $\tau_R{}'(Y)=\{U,\emptyset,c\}$ and $\mu_R{}'(Y)=\{U,\emptyset,\{a\},\{c\},\{a,c\}\}$.Then $\mathfrak{M}_\alpha{}^*(X)$=$\{U,\emptyset,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\}\}$.And $\mathfrak{M}\alpha^{*c}$(X)=$\{U,\emptyset,\{b,c\},\{a,c\}\ ,\{a,b\},\{c\},\{b\},\{a\}\}$.Define by $f(a)=a,f(b)b,f(c)=c$.Hence f is contra $\mathfrak{M}\alpha^*$ - continuous functions.

Theorem : 3.4

Every contra $\mathfrak{M}$ - continuous is contra $\mathfrak{M}\alpha^*$ - continuous functions.

Proof :

Let $f:((U,\tau_R(X),\mu_R(X))\to(V,\tau_R{}'(Y),\mu_R{}'(Y))$ be a contra $\mathfrak{M}$ - continuous.$\Rightarrow f^{-1}(O)$ is $\mathfrak{M}$ - closed in $\tau_R(X)$ for every $\mathfrak{M}$ open set O in $\tau_R(Y)$.Since every $\mathfrak{M}$ closed set is $\mathfrak{M}\alpha^*$ -closed.$\Rightarrow f^{-1}(O)$ is $\mathfrak{M}\alpha^*$- closed in $\tau_R(X)$ $f$or every $\mathfrak{M}$ - open set O in $\tau_R{}'$(Y).$\Rightarrow f$ is contra $\mathfrak{M}\alpha^*$ - continuous functions.

Theorem :3.5

The converse of the above theorem need not true as shown in the following example

Remark :3.6

Let $U=V=\{a,b,c\}$ and $X=\{b,c\}\subseteq U$ with $U/R=\{\{a\},\{b\},\{c\}\}$ and $\mu=\{a\}\notin\tau_R(X)$.Let $\tau_R(X)=\{U,\emptyset,\{b,c\}\}$ and $\mu_R(X)=\{U,\emptyset,\{b,c\},\{a\}\}$ then $\mu_R{}^c$=$\{U,\emptyset,\{b,c\}$,$\{a\}$.Let $\tau_R{}'(Y)=\{U,\emptyset,\{a\},\{b,c\}\}$ and $Y=\{a,c\}\subseteq V$ then $V/R=\{\{a\},\{c\},\{b,d\}\}$ and $\mu=\{a,c\}\notin\tau_R(Y)$.Let $\mu_R{}'(Y)$=$\{U,\emptyset,\{a,c\},\{a,b\},\{a\},\{a,b,c\}\}$.Let $\mathfrak{M}\alpha^*(X)$=$\{U,\emptyset,\{a\},\{b\},\{c\},\{a,b\},\{b,c\}\{a,c\}\}$then $\mathfrak{M}_\alpha{}^{*c}(X)=\{U,\emptyset,\{b,c\},\{a,c\},\{a,b\},\{c\},\{a\},\{b\}\}$.Define $f:\left(U,\tau_R(X),\mu_R(X)\right)\to\left(V,\tau_R{}'(Y),\mu_R{}'(Y)\right)$ by $f(a)=a,f(b)=b,f(c)=c$.Then f is contra $\mathfrak{M}\alpha^*$ continuous functions but not $\mathfrak{M}$ continuous functions. Since for the micro open set $\{\{a,b\},\{a,c\}\}$ are not $\mathfrak{M}$ continuous functions in $\left(V,\tau'_R(Y),\mu_R{}'(Y)\right)$.

Theorem : 3.7

Every contra - $\mathfrak{M}\alpha$ continuous functions is contra - $\mathfrak{M}\alpha^*$ continuous functions.

Proof: Let $f:((U,\tau_R(X),\mu_R(X)) \to (V,\tau_R'(Y),\mu_R'(Y))$ be a contra $\mathfrak{M}\alpha$ - continuous functions.$\Rightarrow f^{-1}(O)$ is $\mathfrak{M}\alpha$- closed in $\tau_R(X)$ for every $\mathfrak{M}\alpha$ open set O in $\tau_R(Y)$.Since every $\mathfrak{M}\alpha$ closed is $\mathfrak{M}\alpha^*$-closed.$\Rightarrow f^{-1}(O)$ is $\mathfrak{M}\alpha^*$ - closed in $\tau_R(X)$ for every $\mathfrak{M}\alpha$ - open set O in $\tau_R(Y)$.$\Rightarrow f$ is contra $\mathfrak{M}\alpha^*$ - continuous functions.

Remark : 3.8

The converse of the above theorem is need not true as shown in the following example.

Example 3.9

Let $U = V = \{a,b,c,d\}$ and $X = \{b,d\}$ with $U/R = \{\{a\},\{b,c\},\{d\}\}$ and $\mu = \{a\} \notin \tau_R(X)$.Let $\tau_R(X) = \{U,\emptyset,\{d\},\{b,c,d\},\{b,c\}\}$and $\mu_R(X) = \{U,\emptyset,\{d\},\{b,c,d\},\{b,c\},\{a\},\{a,d\},\{a,b,c\}\}$.Let $\tau_R'(Y) = \{U,\emptyset,\{b\},\{c,d\},\{b,c,d\}\}$ and $Y = \{b,d\} \subseteq V$ then $V/R = \{\{a\},\{b\},\{c,d\}\}$ and $\mu = \{a,b\} \notin \tau_R(Y)$ then $\mu_R'(Y) = \{U,\emptyset,\{a,c\},\{a,b\},\{a\},\{a,b,c\}\}$.Let $\mathfrak{M}_\alpha{}^*(X) = \{U,\emptyset,\{a\},\{b\},\{c\},\{d\},\{a,b\}\{a,c\},\{a,d\},\{c,b\},\{b,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$then $\mathfrak{M}\alpha^{*c}(X) = \{U,\emptyset,\{b,c,d\},\{a,c,d\},\{a,b,d\},\{a,b,c\},\{c,d\},\{b,d\},\{b,c\},\{a,d\},\{a,c\},\{d\},\{c\}\ \{b\},\{a\}\}$.Let $\mathfrak{M}\alpha(X) = \{\{a\},\{d\},\{a,d\},\{b,c\},\{a,b,c\},\{b,c,d\},U,\emptyset\}$ then $\mathfrak{M}\alpha^c(X) = \{U,\emptyset,\{b,c,d\},\{a,b,c\},\{b,c\},\{a,d\},\{d\},\{a\}\}$.Define $f:(U,\tau_R(X),\mu_R(X)) \to (V,\tau_R'(Y),\mu_R'(Y))$ by $f(a) = b, f(b) = c, f(c) = d, f(d) = a$.Then f is contra $\mathfrak{M}\alpha^*$ - continuous functions but not $\mathfrak{M}\alpha$ - continuous functions. Since for the micro open set $\{a,b,d\}$ is not $\mathfrak{M}\alpha$ - closed in $(V,\tau_R'(Y),\mu_R'(Y))$.

Theorem :3.10

Let arbitrary union of $\mathfrak{M}\alpha^*$ - open Set be micro $\mathfrak{M}\alpha^*$ - open. Then the following statement for a map $f:((U,\tau_R(X),\mu_R(X)) \to (V,\tau_R'(Y),\mu_R'(Y))$.

1) f is contra $\mathfrak{M}\alpha^*$ - continuous.

2)For every $\mathfrak{M}$ - closed set F of V $f^{-1}(F)$ is $\mathfrak{M}\alpha^*$ - open in U.

3)For each $x \in X$ and each micro closed set B of V containing $f(X)$,there exist a that $f(A) \subseteq F$.

4) For each $x \in X$ and each open set B of V not containing x, their exist $\mathfrak{M}\alpha^*$ - closed set K containing x such that $f^{-1}(B) \subseteq K$.

Proof:

(1)⟺(2) obvious.

(2)⇒(3) Let F be micro closed in V containing $f(x)$.Hence $x \in f^{-1}(F)$.By (2) $f^{-1}(F)$ is $\mathfrak{M}\alpha^*$ in U containing X.Let $A = f^{-1}(F)$.This implies A is $\mathfrak{M}\alpha^*$ open in U containing X and $f(A) \subseteq F$.So (3) holds.

(3)⇒(2) Let F be micro closed of V containing $f(x)$.So $x \in f^{-1}(F)$.By (3),there exist $\mathfrak{M}\alpha^*$ open set A_x of U containing x such that $f(A_x) \subseteq F$.So $x \in f^{-1}(F) = U\{A_x : x \in f^{-1}(F)\}$.This is a union of $\mathfrak{M}\alpha^*$- open sets and hence it is $\mathfrak{M}\alpha^*$ - open.Hence (2) holds.

(3)⇒(4) Let B be micro open in V not containing $f(x)$.Then V - B is micro closed in V containing $f(x)$.By (3),there exists $\mathfrak{M}\alpha^*$ - open set A in U containing x such that $f(A) \subseteq V - B$.This implies $A \subseteq f^{-1}(V - B) = U - f^{-1}(B)$.Hence $f^{-1}(B) \subseteq U - A$.Let $K = X - A$. K is $\mathfrak{M}\alpha^*$-closed in U not containing x such that $f^{-1}(B) \subseteq K$.Hence (4) holds.

(4)⇒(3) Let F be micro closed in V containing $f(x)$.Then $V - F$ is micro open in V not containing $f(x)$.From (4),there exists $\mathfrak{M}\alpha^*$-closed K in U not containing x such that $f^{-1}(V - F) \subseteq K$.This implies $U - f^{-1}(F) \subseteq K$.Hence $U - K \subseteq f^{-1}(F)$.That is $f(U - K) \subseteq F$.Let $A = U - K$.Then A is $\mathfrak{M}\alpha^*$-open in U containing x such that $f(A) \subseteq F$.So (3) holds.

Theorem : 3.11

If $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R'(Y), \mu_R'(Y))$ is $\mathfrak{M}\alpha^*$ continuous and $g : (V, \tau_R'(Y), \mu_R'(Y)) \to \left(W, \tau_R''(Z), \mu_R''(Z)\right)$ is contra micro continuous then $gof: (U, \tau_R(X), \mu_R(X)) \to \left(W, \tau_R''(Z), \mu_R''(Z)\right)$ is contra $\mathfrak{M}\alpha^*$- continuous.

Proof :

Let S be a micro open set in$\left(W, \tau_R''(Z), \mu_R''(Z)\right)$.Since g is contra micro continuous we have,$g^{-1}(S)$ is micro closed in $(V, \tau_R'(Y), \mu_R'(Y))$.Since f is $\mathfrak{M}\alpha^*$ continuous and

$g^{-1}(S)$ is micro closed in $(V,\tau_R'(Y),\mu_R'(Y))$ then $f^{-1}(g^{-1}(S) = (gof)^{-1}(S)$ is $\mathfrak{M}\alpha^*$ - closed in $(U,\tau_R(X),\mu_R(X))$.Therefore gof is contra $\mathfrak{M}\alpha^*$ - continuous.

Theorem : 3.12

If $f:(U,\tau_R(X),\mu_R(X)) \to (V,\tau_R'(Y),\mu_R'(Y))$ is contra $\mathfrak{M}\alpha^*$ continuous and $g:(V,\tau_R'(Y),\mu_R'(Y)) \to (W,\tau_R''(Z),\mu_R''(Z))$ is micro continuous then $gof : (U,\tau_R(X),\mu_R(X)) \to (W,\tau_R''(Z),\mu_R''(Z))$ is contra $\mathfrak{M}\alpha^*$ - continuous.

Proof :

Let S be a micro open set in$(W,\tau_R''(Z),\mu_R''(Z))$.Since g is micro continuous then $g^{-1}(S)$ is micro open in $(V,\tau_R'(Y),\mu_R'(Y))$.

Since f is contra $\mathfrak{M}\alpha^*$ continuous and we have $f^{-1}(g^{-1}(S)) = (gof)^{-1}(S)$ is $\mathfrak{M}\alpha^*$ closed in $(U,\tau_R(X),\mu_R(X))$.Therefore gof is contra $\mathfrak{M}\alpha^*$ - continuous.

IV-Contra $\mathfrak{M}\alpha^*$- Irresolute Functions

Definition 4.1

A function $f:(U,\tau_R(X),\mu_R(X)) \to (V,\tau_R'(Y),\mu_R'(Y))$ is called contra $\mathfrak{M}$ - irresolute function if $f^{-1}(S)$ is $\mathfrak{M}$ - closed in $(U,\tau_R(X),\mu_R(X))$ for every $\mathfrak{M}$ - open set S in $(V,\tau_R'(Y),\mu_R'(Y))$.

Definition 4.2

A function $f:(U,\tau_R(X),\mu_R(X)) \to (V,\tau_R'(Y),\mu_R'(Y))$ is called **contra $\mathfrak{M}\alpha^*$-irresolute function** if $f^{-1}(S)$ is $\mathfrak{M}\alpha^*$ closed in $(U,\tau_R(X),\mu_R(X))$ for every $\mathfrak{M}\alpha^*$- open set S in $(V,\tau_R'(Y),\mu_R'(Y))$.

Example 4.3

Let $U = V = \{a,b,c,d\}$ with $U/R = \{\{a\},\{d\},\{b,c\}\}$ and $X = \{b,d\} \subseteq U$ then $\mu = \{\{a\},\{b,c\},\{d\}\}$.Let $\tau_R(X) = \{U,\emptyset,\{d\},\{b,c,d\},\{b,c\}\}$ and $\mu_R(X) = \{U,\emptyset,\{d\},\{b,c,d\},$

$\{b,c\},\{a\},\{a,d\},\{a,b,c\}\}$.Let $Y = \{a,c\} \subseteq V$ with $V/R = \{\{b\},\{c\},\{a,d\}\}$,$\mu = \{a,b\} \notin \tau_R($ Y) then $\mu_R'(Y) = \{U,\emptyset,\{a,c\},\{a,b\},\{a\},\{a,b,c\},\{a,c,d\},\{a,d\}\}$ and $\tau_R'(Y) = \{U,\emptyset,\{c\}\{a,c,d\},\{a,d\}\}$.Let $\mathfrak{M}\alpha^*(X) = \{U,\emptyset,\{a\},\{d\},\{a,d\},\{b,c\},\{a,b,c\},\{b,c,d\}$ Then$\mathfrak{M}\alpha^{*c}(X) = U,\emptyset,\{b,c,d\},\{a,c,d\},\{a,b,d\},\{a,b,c\},\{c,d\},\{b,d\}\{b,c\},\{a,d\},$ $\{a,c\},\{a\},$

$\{b\},\{c\},\{d\}\}$ and $\mathfrak{M}\alpha^*(Y)$=$\{U,\emptyset,\{a\},\{c\},\{a,b\},\{a,c\},\{a,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\}$.Define $f:(U,\tau_R(X),\mu_R(X)) \to (V,\tau_R{}'(Y),\mu_R{}'(Y))$ by $f(a) = c, f(b) = b, f(c) = d, f(d) = a$. Hence f is contra $\mathfrak{M}\alpha^*$ -Irresolute functions.

Theorem 4.4

Every contra $\mathfrak{M}$ irresolute function is contra $\mathfrak{M}\alpha^*$- irresolute functions.

Proof :

Let $f:((U,\tau_R(X),\mu_R(X)) \to (V,\tau_R{}'(Y),\mu_R{}'(Y))$ be a contra $\mathfrak{M}$ - irresolute functions.⇒$f^{-1}(S)$ is $\mathfrak{M}$- closed in $\tau_R(X)$ for every $\mathfrak{M}$ open set S in $\tau_R{}'(Y)$.Since every $\mathfrak{M}$ closed is $\mathfrak{M}\alpha^*$ closed.⇒$f^{-1}(S)$ is $\mathfrak{M}\alpha^*$ - closed in $\tau_R(X)$ for every $\mathfrak{M}\alpha^*$- open set S in $\tau_R(Y)$.⇒f is contra $\mathfrak{M}\alpha^*$- irresolute functions.

Remark :4.5

The converse of the above theorem is need not true as shown in the following example.

Example :4.6

Let $U = V = \{a,b,c,d\}$ with $U\,/\,R = \{\{a\},\{b,c\},\{d\}\}$ and $X = \{b,d\} \subseteq U,\ \mu = \{a\} \notin \tau_R(X)$.Let $\tau_R(X)$ =$\{U,\emptyset,\{d\},\{b,c,d\},\{b,c\}\}$ and $\mu_R(X)$ =$\{U,\emptyset,\{d\},\{b,c,d\},\{b,c\},\{a\},\{a,d\},\{a,b,c\}\}$.Let Y =$\{a,b\}\subseteq V$ with V/R =$\{\{a,b\},\{c\},\{d\}\}$ μ=$\{a,b,c\}\notin \tau_R(Y)$ then

$\tau_R{}'(Y)$ =$\{U,\emptyset,\{a,b\}\}$ and $\mu_R{}'(Y)$ =$\{U,\emptyset,\{a,b\},\{a,b,c\}\}$.Let $\mathfrak{M}^c(X)$ =$\{U,\emptyset,\{b,c,d\},\{a,b,c\},\{b,c\},\{a,d\},\{d\},\{a\}\}$ and $\mathfrak{M}(Y) = U,\emptyset,\{a,b\},\{a,b,c\}\}$.Define $f:(U,\tau_R(X),\mu_R(X)) \to (V,\sigma_R(Y),\mu_R{}'(Y))$ by $f(a) = c, f(b) = b, f(c) = d, f(d) = a$.Since for the $\mathfrak{M}$ open set $\{\{a,b\},\{a,b,c\}\}$ the inverse image is not in $\mathfrak{M}$ closed sets in X.Therefore f is not $\mathfrak{M}$ irresolute.

Theorem 4.7

Every contra $\mathfrak{M}\alpha$ - irresolute function is contra $\mathfrak{M}\alpha^*$ - irresolute functions.

Proof :

Let $f:((U,\tau_R(X),\mu_R(X)) \to (V,\tau_R{}'(Y),\mu_R{}'(Y))$ be a contra $\mathfrak{M}\alpha$ – irresolute functions.⇒$f^{-1}(S)$ is $\mathfrak{M}\alpha$- closed in $\tau_R(X)$ for every $\mathfrak{M}\alpha$ open set S in $\tau_R{}'(Y)$.Since every $\mathfrak{M}\alpha$ - closed is $\mathfrak{M}\alpha^*$ open and $\mathfrak{M}\alpha$ - open is $\mathfrak{M}\alpha^*$ - open.⇒$f^{-1}(S)$ is $\mathfrak{M}\alpha^*$- closed in $\tau_R(X)$ for every $\mathfrak{M}\alpha^*$ - open set S in $\tau_R{}'(Y)$.⇒f is contra $\mathfrak{M}\alpha^*$ - irresolute functions.

Remark :4.8

The converse of the above theorem is need not true as shown in the following example.

Example : 4.9

Let $U = V = \{a,b,c\}$ *with* $U/R = \{\{a\},\{b\},\{c\}\}$ *and* $X = \{b,c\}$ $\subseteq U, \mu$=$\{a\} \notin \tau_R(X)$.

Let $\tau_R(X)$ =$\{U,\emptyset,\{b,c\}\}$and $\mu_R(X)$ =$\{U,\emptyset,\{b,c\},\{a\}\}$ $\mathfrak{M}\alpha^{*c}(X)$ =$\{U,\emptyset,\{b,c\},\{a,c\},\{a,b\},\{c\}$,$\{a\},\{b\}\}$.Let Y =$\{a,b\}$⊆V ,$V/R = \{\{a\},\{b,c\}\}$ *and* $\mu = \{b\} \notin \tau_R(Y)$ $\mu_R{}'(Y)$ =$\{U,\emptyset,\{a\}$,{b,c}, $\{b\},\{a,b\}\}$ and $\tau_R{}'(Y)$ =$\{U,\emptyset,\{a\},\{b,c\}$.Let $\mathfrak{M}_\alpha(X)$ =$\{U,\emptyset,\{a\},\{b,c\}\}$ then $\mathfrak{M}_\alpha{}^c(X) = \{U,\emptyset$,$\{b,c\},\{a\}$ $\}$and $\mathfrak{M}^*(Y) = \{U,\emptyset,\{a\},\{b\},\{a,b\}\{b,c\}\}$. Define $f: U,\tau_R(X),\mu_R(X)) \to (V,\tau_R{}'(Y),\mu_R{}'(Y))$ by $f(a) = a, f(b) = b, f(c) = c$.Then f is contra $\mathfrak{M}\alpha^*$ irresolute function but not $\mathfrak{M}\alpha$ irresolute functions. Since for the micro α - open set $\{\{b\},\{a,b\}\}$ is not $\mathfrak{M}\alpha$ irresolute functions in $(V,\tau_R{}'(Y),\mu_R{}'(Y))$.

Theorem : 4.10

Let $f: (U,\tau_R(X),\mu_R(X)) \to (V,\tau_R{}'(Y),\mu_R{}'(Y))$ and $g: (V,\tau_R{}'(Y),\mu_R{}'(Y)) \to (W,\tau_R{}''(Z),\mu_R{}''(Y))$ be to functions.

1)If f is contra $\mathfrak{M}\alpha^*$ irresolute and g is $\mathfrak{M}\alpha^*$ irresolute then gof is contra $\mathfrak{M}\alpha^*$ irresolute.

2)If f is contra $\mathfrak{M}\alpha^*$ irresolute and g is $\mathfrak{M}\alpha^*$ continuous then gof is contra $\mathfrak{M}\alpha^*$ continuous.

3)If f is contra $\mathfrak{M}\alpha^*$ irresolute and g is $\mathfrak{M}$ continuous then gof is contra $\mathfrak{M}\alpha^*$ continuous.

4)If f is contra $\mathfrak{M}\alpha^*$ irresolute and g is $\mathfrak{M}\alpha$ continuous then gof is contra $\mathfrak{M}\alpha^*$- continuous.

Proof :

1)Let S be a $\mathfrak{M}\alpha^*$ open set in $(W,\tau_R{}''(Z),\mu_R{}''(Z))$.Since g is $\mathfrak{M}\alpha^*$ irresolute and S is $\mathfrak{M}\alpha^*$ open set,$g^{-1}(S)$ is $\mathfrak{M}\alpha^*$ open in $(V,\tau_R{}'(Y),\mu_R{}'(Y))$.Since f is contra $\mathfrak{M}\alpha^*$ irresolute and $g^{-1}(S)$ is $\mathfrak{M}\alpha^*$ open in $(V,\tau_R{}'(Y),\mu_R{}'(Y))$,$f^{-1}(g^{-1}(S)) = (gof)^{-1}(S)$ is $\mathfrak{M}\alpha^*$ closed in $(U,\tau_R(A),\mu_R(A))$.Therefore gof is contra $\mathfrak{M}\alpha^*$ - irresolute functions.

2)Let S be micro open set in $(W,\tau_R{}''(Z),\mu_R{}''(Z))$.Since g is $\mathfrak{M}\alpha^*$ continuous and S is micro open set ,$g^{-1}(S)$ is $\mathfrak{M}\alpha^*$ open in ($V,\tau_R{}'(Y),\mu_R{}'(Y))$.Since f is contra $\mathfrak{M}\alpha^*$ irresolute and $g^{-1}(S)$ is $\mathfrak{M}\alpha^*$ open in $(V,\tau_R'(Y),\mu_R'(Y)), f^{-1}(g^{-1}(S)) = (gof)^{-1}(S)$ is $\mathfrak{M}\alpha^*$ closed in $(U,\tau_R(X))$.Therefore gof is contra $\mathfrak{M}\alpha^*$ continuous functions.

3)Let S be micro open set in $\left(W,\tau_R''(Z),\mu_R''(Z)\right)$.Since g is $\mathfrak{M}$ continuous and S is micro open set $g^{-1}(S)$ is micro open in $\left(V,\tau_R'(Y),\mu_R'(Y)\right).\Rightarrow g^{-1}(S)$is $\mathfrak{M}\alpha^*$open $\left(V,\tau_R'(Y),\mu_R'(Y)\right)$. since every micro open set is $\mathfrak{M}\alpha^*$ is open. Since f is contra $\mathfrak{M}\alpha^*$ irresolute and $g^{-1}(S)$ is $\mathfrak{M}\alpha^*$ open in $\left(V,\tau_R'(Y),\mu_R'(Y)\right), f^{-1}(g^{-1}(S)) = (gof)^{-1}(S)$ is $\mathfrak{M}\alpha^*$ closed in $\left(U,\tau_R(X),\mu_R(X)\right)$.Therefore gof is contra $\mathfrak{M}\alpha^*$ continuous functions.

4) Let S be micro open set in $(W,\tau_R''(Z),\mu_R''(Z))$.Since g is $\mathfrak{M}\alpha$ continuous and S is micro open set $g^{-1}(S)$ is $\mathfrak{M}\alpha$ open in $(V,\tau_R'(Y),\mu_R'(Y)).\Rightarrow g^{-1}(S)$ is $\mathfrak{M}\alpha^*$ open in $(V,\tau_R'(Y),\mu_R'(Y))$. since every micro open set is $\mathfrak{M}\alpha^*$ is open.Since f is contra $\mathfrak{M}\alpha^*$ irresolute and $g^{-1}(S)$ is $\mathfrak{M}\alpha^*$ open in $(V,\tau_R'(Y),\mu_R'(Y)), f^{-1}(g^{-1}(S)) = (gof)^{-1}(S)$ is $\mathfrak{M}\alpha^*$ closed in $(U,\tau_R(X),\mu_R(X))$.Therefore gof is contra $\mathfrak{M}\alpha^*$ continuous functions.

CONCLUSION

In this paper, we have introduced Contra Micro α^*- continuous functions and Contra Micro α^* - Irresolute functions and explored their fundamental properties. We have examined their relationships with varies existing Micro Continuous and Micro Irresolute functions. Furthermore, several compositions of these functions have been investigated, provided deeper insights into their structural behaviour. These findings contribute to the ongoing study of function spaces in Micro topological settings.

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