CONTRA CONTINUOUS AND CONTRA IRRESOLUTE FUNCTIONS VIA NANO PRE*- OPEN SETS

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ABSTRACT: The purpose of this paper is to introduce and study a new class of functions called contra nano pre*- continuous and contra nano pre*- irresolute functions in nano topological spaces and investigate some of its properties.

KEY WORDS: Nano Pre*- Open, Nano Pre*- Closed, Contra Nano Pre*- Continuous, Contra Nano Pre*- Irresolute.

I INTRODUCTION

Lellis Thivagar et al. [2] introduced nano topological space with respect to a subset X of an universe which is defined in terms of lower and upper approximations of X. The elements of a nano topological space are called the nano open sets. He also introduced the weak forms of nano open sets namely nano α - open sets, nano semi -open sets, nano regular - open sets and nano pre - open sets. In 2014, K. Bhuvaneswari and K. Mythili Gnanapriya[1] introduced Nano Generalised Closed Sets in Nano Topological Space. Recently, the authors C. Reena and R.Raxi and S. Bhuvaneswari [3] introduced a new class of nano sets namely, nano pre*- open sets in nano topological spaces. This paper analyzes nano pre*- open sets to introduce and examine the ideas of contra nano pre*- continuous and contra nano pre*- irresolute functions in nano topological spaces.

II PRELIMINARIES

Throughout this paper $(U, \tau_R(X)), (V, \sigma_R(Y)), (W, \eta_R(Z))$ represent non empty nano topological spaces on which no separation axioms are assumed unless otherwise mentioned.

Definition 2.1[1]: Let $(U, \tau_R(X))$ be a Nano topological space. A subset A of $(U, \tau_R(X))$ is called ***Nano generalized - closed set*** if $Ncl(A) \subseteq V$ where $A \subseteq V$ and V is Nano - open. The complement of Nano generalized - closed set is called as ***Nano generalized - open set***.

Definition 2.2[1]: For every set $A \subseteq U$,

(i) the ***Nano generalized closure of A*** is defined as the intersection of all Ng - closed sets containing A and is denoted by $Ncl^*(A)$.

(ii) the ***Nano generalized interior of A*** is defined as the union of all Ng - open sets contained in A and is denoted by $Nint^*(A)$.

Definition 2.3[3]: A subset A of a Nano topological space $(U, \tau_R(X))$ is called a ***Nano pre*-open*** set if $A \subseteq Nint^*(Ncl(A))$.

Definition 2.4[3]: The complement of a Nano pre*-open set is called ***Nano pre*-closed.***

Theorem 2.5[3]: (i) Every nano - closed set is nano pre*- closed.

(ii) Every nano pre - closed set is nano pre*- closed.

(iii) Every nano α - closed set is nano pre*- closed.

(iv) Every nano regular - closed set is nano pre*- closed.

(v) Every nano g - closed set is nano pre*- closed.

Definition 2.7: A function f: (U,τ_R(X)) → (V,σ_R(Y)) is said to be

(i) contra nano continuous if $f^{-1}(V)$ is nano closed in (U,τ_R(X)) for every nano open set V in (Y,σ_R(Y)).

(ii) contra nano regular*- continuous if $f^{-1}(V)$ is nano regular*- closed in (U,τ_R(X)) for every nano open set V in (Y,σ_R(Y)).

(iii) contra nano regular - continuous if $f^{-1}(V)$ is nano regular - closed in (U,τ_R(X)) for every nano open set V in (Y,σ_R(Y)).

(iv) contra nano α - continuous if $f^{-1}(V)$ is nano α - closed in (U,τ_R(X)) for every nano open set V in (Y,σ_R(Y)).

(v) contra nano semi*- continuous if $f^{-1}(V)$ is nano semi*- closed in (U,τ_R(X)) for every nano open set V in (Y,σ_R(Y)).

III CONTRA NANO PRE*-CONTINUOUS FUNCTIONS

Definition 3.1: A function $f : (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$ is said to be **Contra Nano Pre*-Continuous** if $f^{-1}(V)$ is Nano Pre*- closed in $(U, \tau_R(X))$ for every nano open set V in $(V, \sigma_R(Y))$.

Example 3.2 : Let U={a,b,c,d} , U/R={{a,b},{c,d}} and X={a,c,d}. Then τ_R(X)= {U,∅,{a,b},{c,d}},NP*O(U,τ_R(X)) = {U,∅,{a},{b},{c},{d},{a,b},{a,c},{a,d}, {b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}},NP*C(U,τ_R(X))={∅,U,{b,c,d}, {a,c,d},{a,b,d},{a,b,c},{c,d},{b,d},{b,c},{a,d},{a,c},{a,b},{d},{c},{b},{a}}. Let V = {a,b,c,d},V/R = {{b},{c},{d}} and Y={a,b,c}.Then σ_R(Y)={V,∅,{b,c}}. Define $f : (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$ by $f(a) = b$; $f(b) = c$; $f(c) = d$; $f(d) = a$.Then $f^{-1}\{b, c\}$=$\{a, b\}$ is contra nano pre*- closed in $(U, \tau_R(X))$. Hence f is contra nano pre*-continuous.

Theorem 3.3: Every contra nano continuous function is contra nano pre*-continuous.

Proof: Let $f: (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$ be contra nano continuous. Let V be any nano open set in $(V, \sigma_R(Y))$. Then $f^{-1}(V)$ is nano closed in $(U, \tau_R(X))$. Hence by theorem 2.5(i), $f^{-1}(V)$ is nano pre*- closed in $(U, \tau_R(X))$. Therefore f is contra nano pre*-continuous.

Remark 3.4: The converse of the above theorem is not true as shown in the following example.

Example 3.5: Let U={a,b,c},U/R={{a},{b},{c}} and X={b,c}. Then τ_R(X)={U,∅,{b,c}},NC(U,τ_R(X)) = {U,∅,{a}}, NP*O(U,τ_R(X)) = {U,∅,{b},{c},{a,b}, {a,c},{b,c}}, NP*C(U,τ_R(X)) = {U,∅,{a,c},{a,b},{c},{b},{a}. Let V = {a,b,c,d}, V/R = {{a},{b}} and Y={a,c}. Then σ_R(Y) = {V,∅,{a}}. Define $f: (U,\tau_R(X)) \rightarrow (V,\sigma_R(Y))$ by $f(a) = b$; $f(b) = a$; $f(c) = c$. Then f is conta nano pre*-continuous but not contra nano continuous. Since for the nano open set $\{b, c\}$ in $(V, \sigma_R(Y))$, $f^{-1}\{a\} = \{b\}$ is nano pre*- closed but not nano closed in $(U, \tau_R(X))$.

Theorem 3.6: Every contra nano pre-continuous function is contra nano pre*-continuous.

Proof: Let $f: (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$ be contra nano pre - continuous. Let V be any nano open set in $(V, \sigma_R(Y))$. Then $f^{-1}(V)$ is nano pre - closed in $(U, \tau_R(X))$. Hence by theorem 2.5(ii),$f^{-1}(V)$ is nano pre*- closed in $(U, \tau_R(X))$. Therefore f is contra nano pre*-continuous.

Remark 3.7: The converse of the above theorem is not true as shown in the following example.

Example 3.8: Let $U = \{a, b, c, d\}, U/R = \{\{a\}, \{b\}\}$ and $X = \{a, c, d\}$.Then $\tau_R(X)$ ={U,∅,{a}}, NPO $((U,\tau_R(X))$ = {U,∅,{a},{a,b},{a,c},{a,d},{a,b,c},{a,b,d}, {a,c,d}}, NPC$((U,\tau_R(X))$ = {U,∅,{b,c,d},{c,d},{b,d},{b,c},{d},{c},{b}}, NP*O$(U,\tau_R(X))$ = {U,∅,{a},{b},{c},{d},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c}, {a,b,d},{a,c,d},{b,c,d}}, NP*C$(U,\tau_R(X))$={U,∅,{b,c,d},{a,c,d},{a,b,d},{a,b,c},{c,d}, {b,d},{b,c},{a,d},{a,c},{a,b},{d},{c},{b},{a}}.Let V={a,b,c,d}, V/R={{b},{c},{d}} and Y={a,b,c}. Then $\sigma_R(Y)$= {V,∅,{b,c}}. Define $f: (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$ by f(a)= b; f(b)= c;f(c) = d; f(d) = a. Then f is contra nano pre*- continuous but not contra nano pre - continuous. Since for the nano open set $\{b, c\}$ in $(V, \sigma_R(Y), f^{-1}\{b, c\} = \{a, b\}$ is nano pre*-closed but not nano pre - closed in $(U, \tau_R(X))$.

Theorem 3.9: Every contra nano α - continuous function is contra nano pre*-continuous.

Proof: Let $f: (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$ be contra nano α - continuous. Let V be any nano open set in $(V, \sigma_R(Y))$. Then $f^{-1}(V)$ is nano α - closed in $(U, \tau_R(X))$. Hence by theorem 2.5(iii),$f^{-1}(V)$ is nano pre*- closed in $(U, \tau_R(X))$. Therefore f is contra nano pre*-continuous.

Remark 3.10: The converse of the above theorem is not true as shown in the following example.

Example 3:11: Let U = {a,b,c,d}, U/R = {{a},{b},{c},{d}} and X = {a,b,c}. Then τ_R(X) = {U,∅,{a,b,c}}, NαO(U,τ_R(X)) = {U,∅,{a,b,c}}, NαC(U,τ_R(X)) = {U,∅,{d}}, NP*O(U,τ_R(X))={U,∅,{a},{b},{c},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c}, {a,b,d},{a,c,d},{b,c,d}}, NP*C(U,τ_R(X)) ={U,∅,{b,c,d},{a,c,d},{a,b,d},{c,d}, {b,d},{b,c},{a,d},{a,c},{a,b},{d},{c},{b},{a}}. Let V={a,b,c,d},V/R={{b},{c},{d}} and Y={a,b,c}. Then σ_R(Y) = {V,∅,{b,c}}. Define $f: (U,\tau_R(X)) \to (V,\sigma_R(Y))$ by $f(a) = b$; $f(b) = c$; $f(c) = d$; $f(d) = a$. Then f is contra nano pre*-continuous but not contra nano α - continuous. Since for the nano open set $\{b,c\}$ in $(V,\sigma_R(Y))$, $f^{-1}\{b,c\} = \{a,b\}$ is nano pre*- closed but not nano α - closed in $(U,\tau_R(X))$.

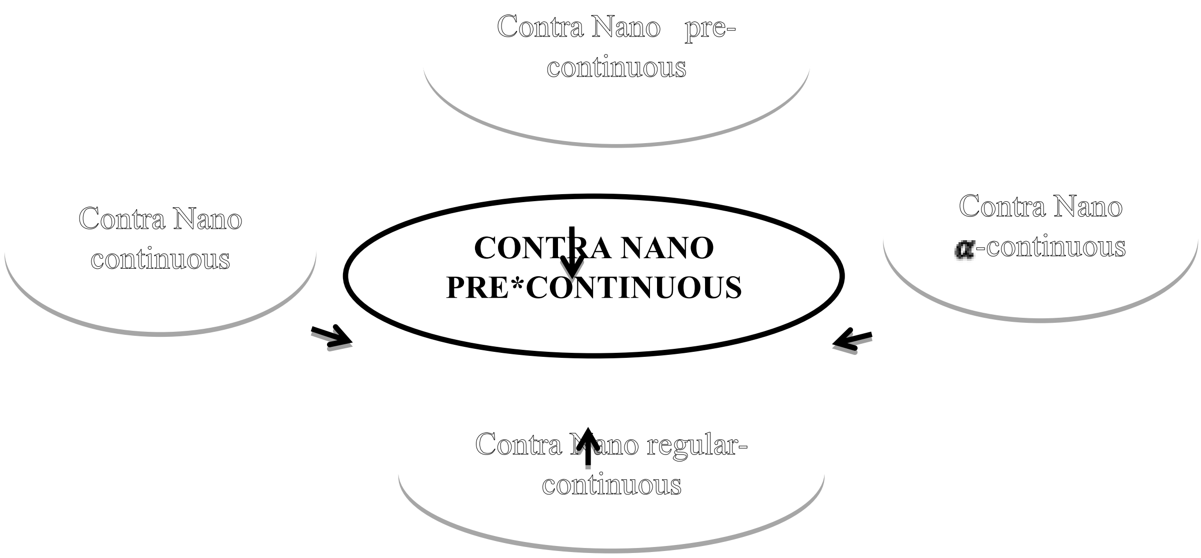
Theorem 3.12: Every contra nano regular - continuous function is contra nano pre*-continuous.

Proof: Let $f: (U,\tau_R(X)) \to (V,\sigma_R(Y))$ be contra nano regular - continuous. Let V be any nano open set in $(V,\sigma_R(Y))$. Then $f^{-1}(V)$ is nano regular - closed in $(U,\tau_R(X))$. Hence by theorem 2.5(iv), $f^{-1}(V)$ is nano pre*- closed in $(U,\tau_R(X))$. Therefore f is contra nano pre*-continuous.

Remark 3.13: The converse of the above theorem is not true as shown in the following example.

Example 3.14: Let $U = \{a,b,c,d\}$, $U/R = \{\{a,b\},\{c,d\}\}$ and $X = \{a,c,d\}$. Then τ_R(X) = {U,∅,{c,d},{a,b}}, NRO(U,τ_R(X)) = {U,∅,{a,b},{c,d}},NRC(U,τ_R(X)) = {U,∅,{c,d},{a,b}},NP*O(U,τ_R(X))={U,∅,{a},{b},{c},{d},{a,b},{a,c},{a,d},{b,c}, {b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}},NP*C(U,τ_R(X)) = {U,∅,{b,c,d},{a,c,d}, {a,b,d},{a,b,c},{c,d},{b,d},{b,c},{a,d},{a,c},{a,b},{d},{c},{b},{a}}.Let V={a,b,c,d}, V/R={{b},{c},{d}} and Y={a,b,c}. Then $\sigma_R(Y) = \{V,\emptyset,\{b,c\}\}$. Define $f: (U,\tau_R(X)) \to (V,\sigma_R(Y))$ by $f(a) = d$; $f(b) = b$; $f(c) = a$; $f(d) = c$. Then f is contra nano pre*- continuous but not nano regular - continuous. Since for the nano open set $\{b,c\}$ in $(V,\sigma_R(Y))$, $f^{-1}\{b,c\} = \{b,d\}$ is nano pre*-closed but not nano regular - closed in $(U,\tau_R(X))$.

From the above discussion we have the following diagram:



Theorem 3.15: Let $(U, \tau_R(X))$ and $(V, \sigma_R(Y))$ be two nano topological spaces. Then $f: (\mathrm{U}, \tau_R(\mathrm{X})) \to (\mathrm{V}, \sigma_R(\mathrm{Y}))$ is contra nano pre*- continuous if and only if the inverse image of every nano closed set in V is nan pre*- open in U.

Proof: Let $f: (\mathrm{U}, \tau_R(\mathrm{X})) \to (\mathrm{V}, \sigma_R(\mathrm{Y}))$ be a contra nano pre*- continuous and S be a nano closed set in $(\mathrm{V}, \sigma_R(\mathrm{Y}))$. Since f is contra nano pre*- continuous function, $f^{-1}(S^c) = (f^{-1}(S))^c$ is nano pre*-closed in $(U, \tau_R(X))$. Hence $f^{-1}(S)$ is nano pre*- open set in $(U, \tau_R(X))$. Conversely, suppose the inverse image of every nano closed set in $(V, \sigma_R(Y))$ is nano pre*- open in $(U, \tau_R(X))$. Let S be a nano open set in $(V, \sigma_R(Y))$. Then S^c is nano closed in $(V, \sigma_R(Y))$. By assumption, $f^{-1}(S^c) = (f^{-1}(S))^c$ is nano pre*- open in $(U, \tau_R(X))$. Hence $f$is contra nano pre*- continuous.

Definition 3.16: A nano topological space $(\mathrm{U}, \tau_R(\mathrm{X}))$ is said to be a nano $\boldsymbol{T_{p^*}}$ **space** if every nano pre*- open set is nano open.

Theorem 3.17: Let $f: (\mathrm{U}, \tau_R(\mathrm{X})) \to (\mathrm{V}, \sigma_R(\mathrm{Y}))$ be a contra nano pre*- continuous function and $(\mathrm{U}, \tau_R(\mathrm{X}))$ be a nano T_{p^*}space. Then f is contra nano continuous.

Proof: Let S be a nano closed set in $(V, \sigma_R(Y))$. Since f is contra nano pre*- continuous, $f^{-1}(S)$ is nano pre*- open set in $(U, \tau_R(X))$. Now since $(U, \tau_R(X))$ is nano T_{p^*} space, $f^{-1}(S)$ is nano open in $(U, \tau_R(X))$. Therefore f is contra nano continuous.

Theorem 3.18: If a function $f: (U, \tau_R(X)) \to (V, \sigma_R(Y))$ is nano pre*- continuous and if $(V, \sigma_R(Y))$ is locally indiscrete, then f is contra nano pre*- continuous.

Proof: Let G be an nano open set $(V, \sigma_R(Y))$. Since $(V, \sigma_R(Y))$ is locally indiscrete, G is nano closed in $(V, \sigma_R(Y))$. Since f is nano pre*- continuous, $f^{-1}(G)$ is nano pre*-closed in $(U, \tau_R(X))$ .Therefore f is contra nano pre*- continuous.

Remark 3.19: The composition of two contra nano pre*- continuous functions need not be contra nano pre*- continuous as seen from the following example.

Example 3.20: Let U = {a,b,c,d},U/R = {{a},{b}} and X = {a,c,d}. Then $\tau_R(X)$={U,ϕ,{a}},NP*O(U,τ_R(X)={U,ϕ,{a},{a,b},{a,c},{a,d},{a,b,c},{a,b,d},{a,c,d},NP* C(U,τ_R(X)={U,ϕ,{b,c,d},{c,d},{b,d},{b,c},{d},{c},{a}}. Let V = {a,b,c,d}, V/R = {{b},{c},{d}} and Y = {a,b,c}.Then $\sigma_R(Y)$={V,ϕ,{b,c}}, NP*O(V,σ_R(Y)) = {V,ϕ,{a},{b},{c},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}},NP*C(V,σ_R(Y))={V,ϕ,{b,c,d},{a,c,d},{a,b,d},{c,d},{b,d},{b,c},{a,d},{a,c},{a,b}, {d},{c},{b},{a}. Let W={a,b,c,d},W/R={{a},{b}}and Z={a,c,d}.Then $\eta_R(Z)$={W,ϕ,{a}},NP*O(W,η_R(Y))={W,ϕ,{a},{b},{c},{d},{a,b},{a,c},{a,d},{b,c}, {b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}},NP*C(V,σ_R(Y))={W,ϕ,{b,c,d},{a,c,d}, {a,b,d},{a,b,c},{c,d},{b,d},{b,c},{a,d},{a,c},{a,b},{d},{c},{b},{a}}. Define $f: (U, \tau_R(X)) \to (V, \sigma_R(Y))$ by $f(a) = a$; $f(b) = c$; $f(c) = b$; $f(d) = d$ and define $g: (V, \sigma_R(Y)) \to (W, \eta_R(z))$ by $g(a) = a$; $g(b) = b$; $g(c) = c$; $g(d) = d$. Then f and g are contra nano pre*- continuous but g ∘ f is not contra nano pre*- continuous. Since $\{a\}$ is nano open in $(W, \eta_R(z))$ but $(g \circ f)^{-1}(\{a\}) = f^{-1}(g^{-1}\{a\}) = f^{-1}(\{a\}) = \{a\}$ which is not nano pre*- closed in $(U, \tau_R(X))$.

Theorem 3.21: Let $f:(\mathrm{U},\tau_R(\mathrm{X})) \rightarrow (\mathrm{V},\sigma_R(\mathrm{Y}))$ be nano pre*- continuous and $g:(V,\sigma_R(Y) \rightarrow (W,\eta_R(Z))$ be contra nano pre*- continous function and let $(\mathrm{V},\sigma_R(\mathrm{Y}))$ be a T_{p^*} space. Then $(g \circ f):(\mathrm{U},\tau_R(\mathrm{X})) \rightarrow (\mathrm{W},\eta_R(\mathrm{Z}))$ is contra nano pre*-continuous.

Proof: Let S be nano closed set in $(\mathrm{W},\eta_R(\mathrm{Z}))$. Since g is contra nano pre*-continuous, $g^{-1}(S)$ is nano pre*- open in $(\mathrm{V},\sigma_R(\mathrm{Y}))$. Now since $(\mathrm{V},\sigma_R(\mathrm{Y}))$ is a T_{p^*} space, $g^{-1}(S)$ is nano open in $(V,\sigma_R(Y))$. Also $f$is nano pre*- continuous, $f^{-1}(g^{-1}(S))$=$(g \circ f)^{-1}(S)$ is nano pre*- open in $(\mathrm{U},\tau_R(\mathrm{X}))$. Hence by theorem 3.15, $g \circ f$ is contra nano pre*- continuous.

Theorem 3.22: Let $f:(\mathrm{U},\tau_R(\mathrm{X})) \rightarrow (\mathrm{V},\sigma_R(\mathrm{Y}))$ be nano pre*- irresolute and $g:(V,\sigma_R(Y) \rightarrow (W,\eta_R(Z))$ be contra nano pre*- continous function. Then $g \circ f:(\mathrm{U},\tau_R(\mathrm{X})) \rightarrow (\mathrm{W},\eta_R(\mathrm{Z}))$ is contra nano pre*- continuous.

Proof: Let G be nano open set in $(W,\eta_R(\mathrm{Z}))$. Since g is contra nano pre*-continuous, $g^{-1}(G)$ is nano pre*- closed in $(\mathrm{V},\sigma_R(\mathrm{Y}))$. Also, since f is nano pre*-irresolute, $f^{-1}(g^{-1}(G))$=$(g \circ f)^{-1}(G)$ is nano pre*- closed in $(\mathrm{U},\tau_R(\mathrm{X}))$. Hence $g \circ f$ is contra nano pre*- continuous.

IV CONTRA NANO PRE*-IRRESOLUTE FUNCTION

Definition 4.1: Let $(\mathrm{U},\tau_R(\mathrm{X}))$ and $(\mathrm{V},\sigma_R(\mathrm{Y})$ be two nano topological spaces. A function $f:(U,\tau_R(X)) \rightarrow (V,\sigma_R(Y))$ is said to be ***contra nano pre*- irresolute*** if $f^{-1}(V)$ is nano pre*- closed in $(U,\tau_R(X))$ for every nano pre*- open set V in $(V,\sigma_R(Y))$.

Example 4.2: Let U = {a,b,c}, U/R = {{c},{a,b}} and X = {a,b,c}. Then $\tau_R(X)$ = {U,∅}, NP*O(U,τ_R(X)) = {U,∅,{a}, {b},{c},{a,b},{a,c},{b,c}}, NP*C(U,τ_R(X)) = {U,∅,{a},{b},{c},{a,b},{a,c},{b,c}}. Let V = {a,b,c}, V/R = {{a},{c}} and Y = {a,b,c}.Then σ_R(Y) = {V,∅,{a,c}},NP*O(V,σ_R(Y)) = {V,∅,{a},{c},{a,b},{a,c},{b,c}}. Define f : (U,τ_R(X)) → (V,σ_R(Y)) by $f(a) = c$; $f(b) = a$; $f(c) = b$. Then f^{-1}{a} = {b}, f^{-1}{b} = {c}, f^{-1}{c} = {a}, f^{-1}{a,b} = {b,c}, f^{-1}{a,c} = {a,b} , f^{-1}{b,c} = {a,c} are nano pre*- closed in $(U,\tau_R(X))$. Hence f is contra nano pre*- irresolute function.

Theorem 4.3: A function $f: (\mathrm{U}, \tau_R(\mathrm{X})) \to (\mathrm{V}, \sigma_R(\mathrm{Y}))$ is contra nano pre*- irresolute if and only if the inverse image of every nano pre*- closed set in $(\mathrm{V}, \sigma_R(\mathrm{Y}))$ is nano pre*-open in $(\mathrm{U}, \tau_R(\mathrm{X}))$.

Proof: Let $f: (\mathrm{U}, \tau_R(\mathrm{X})) \to (\mathrm{V}, \sigma_R(\mathrm{Y}))$ be a contra nano pre*-irresolute and S be nano pre*-closed set in $(\mathrm{V}, \sigma_R(\mathrm{Y}))$. Since f is contra nano pre*-irresolute function, $f^{-1}(S^c) = (f^{-1}(S))^c$ is nano pre*- closed in $(U, \tau_R(X))$. Hence $f^{-1}(S)$ is nano pre*-open set in $(U, \tau_R(X))$. Conversely, suppose the inverse image of every nano pre*-closed set in $(V, \sigma_R(Y))$ is nano pre*- open in $(U, \tau_R(X))$. Let S be a nano pre*- open set in $(V, \sigma_R(Y))$. Then S^c is nano pre*- closed in $(V, \sigma_R(Y))$. By assumption, $f^{-1}(S^c) = (f^{-1}(S))^c$ is nano pre*- open in $(\mathrm{U}, \tau_R(\mathrm{X}))$. Hence $f$is contra nano pre*- irresolute.

Theorem 4.4: Let $f: (\mathrm{U}, \tau_R(\mathrm{X})) \to (\mathrm{V}, \sigma_R(\mathrm{Y}))$ be nano pre*- irresolute and $g:(V, \sigma_R(Y) \to (W, \eta_R(Z)$ be contra nano pre*- irresolute. Then $(g \circ f): (\mathrm{U}, \tau_R(\mathrm{X})) \to (W, \eta_R(Z))$ is contra nano pre*- irresolute.

Proof: Let G be nano pre*-open set in $(W, \eta_R(Z))$. Since g is contra nano pre*- irresolute, $g^{-1}(G)$ is nano pre*- closed in $(\mathrm{V}, \sigma_R(\mathrm{Y}))$. Since $f$is nano pre*- irresolute,$(g \circ f)^{-1}(G) = f^{-1}(g^{-1}(G))$ is nano pre*- closed in $(\mathrm{U}, \tau_R(\mathrm{X}))$. Hence $g \circ f$ is contra nano pre*- irresolute.

Theorem 4.5: Let $f: (\mathrm{U}, \tau_R(\mathrm{X})) \to (\mathrm{V}, \sigma_R(\mathrm{Y}))$ be contra nano pre*-irresolute and $g:(V, \sigma_R(Y) \to (W, \eta_R(Z)$ be nano pre*- irresolute. Then $(g \circ f): (\mathrm{U}, \tau_R(\mathrm{X})) \to (W, \eta_R(Z)$ is contra nano pre*- irresolute.

Proof: Let G be a nano pre*- open set in $(W, \eta_R(Z))$.Since g is nano pre*- irresolute, $g^{-1}(G)$ is nano pre*- open in $(\mathrm{V}, \sigma_R(\mathrm{Y}))$. Since $f$is contra nano pre*- irresolute, $(g \circ f)^{-1}(G) = f^{-1}(g^{-1}(G))$ is nano pre*- closed in $(\mathrm{U}, \tau_R(\mathrm{X}))$. Hence $g \circ f$ is contra nano pre*- irresolute.

Theorem 4.6: If $f: (\mathrm{U}, \tau_R(\mathrm{X})) \to (\mathrm{V}, \sigma_R(\mathrm{Y}))$ and $g:(V, \sigma_R(Y) \to (W, \eta_R(Z)$ are contra nano pre*- irresolute functions, then $g \circ f: (\mathrm{U}, \tau_R(\mathrm{X})) \to (W, \eta_R(Z)\ )$ is nano pre*- irresolute.

Proof: Let G be nano pre*- open set in Z. Since g is contra nano pre*- irresolute, $g^{-1}(G)$ is nano pre*- closed in $(V, \sigma_R(Y))$. Also, since f is contra nano pre*-irresolute, by theorem 4.3, $(g \circ f)^{-1}(G) = f^{-1}(g^{-1}(G))$ is nano pre*- open in $(U, \tau_R(X))$. Hence $g \circ f$ is nano pre*- irresolute.

V CONCLUSION

In this paper, we have introduced contra nano pre*- continuous and contra nano pre*- irresolute functions and investigated their fundamental properties. Also we have examined their relationships with other existing nano continuous and nano irresolute functions.

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