CONTRA FUNCTIONS USING NANO $g\alpha^*$ - CLOSED SETS IN NANO TOPOLOGICAL SPACES

K. Gomathi Ambigai

Department of Mathematics, St. Mary's College (Autonomous) Thoothukudi

Affiliated to Manonmaniam Sundaranar University, Tirunelveli, Tamilnadu, India.

Abstract: The aim of this paper is to introduce and study the concept of new class of functions called Contra Nano $g\alpha^*$-continuous function and Contra Nano $g\alpha^*$-irresolute function in Nano topological spaces.

Keywords : $Ng\alpha^*$ – open, $Ng\alpha^*$ - closed , contra $Ng\alpha^*$ -continuous functions, contra $Ng\alpha^*$ - irresolute functions.

I. INTRODUCTION

Continuity and irresoluteness are fundamental concepts in topology, playing a crucial role in understanding the behaviour of functions between topological spaces. Several generalized forms of continuity have been introduced to study various properties of topological functions. The concept of Nano topology was first introduced by M. Lellis Thivagar [5] and Carmel Richard, defining topological structures in terms of approximations and boundary regions using equivalence relations. They introduced key notions such as Nano closed sets, Nano interior, Nano closure, Nano continuous functions, Nano open mappings, Nano closed mappings and Nano homeomorphisms. K. Bhuvaneswari.K [3] and Mythili Gnanapriya. K introduced Nano g-closed. Later Bhuvaneshwari. K [4] and Ezhilarasi. K introduced Nano semi-generalized and Nano generalized semi- closed sets. P. Subithra [1]and P. Anbarasi Rodrigo introduced $Ng\alpha^*$-closed sets in Nano topological spaces. Later P. Subithra [2]and P. Anbarasi Rodrigo introduced $Ng\alpha^*$-continuous and $Ng\alpha^*$- irresolute in nano topological spaces. This paper aims to introduce and investigate a new class of functions called contra $Ng\alpha^*$continuous functions and contra $Ng\alpha^*$ irresolute functions in Nano topological spaces. The fundamental properties and characterizations of these functions are analyzed, establishing their significance in the broader framework of Nano topology.

II. PRELIMINARIES

In this section, we recall some basic definitions and results in Nano topological spaces which are useful in the sequel.

Definition 2.1 :[5] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq_N U$. Then

1) **The lower approximation of X** with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$.

$L_R(X) = \bigcup_{x \in U}\{R(x): R(x) \subseteq_N X\}$

2) **The upper approximation of X** with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$

$U_R(X) = \bigcup_{x \in U}\{R(x): R(x) \cap X \neq \emptyset\}$

3) The **boundary region of X** with respect to R is the set of all objects, which can be classified neither as X nor as not –X with respect to R and it is denoted by $B_R(X)$.

$B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2 :[5] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \emptyset, U_R(X), L_R(X), B_R(X)\}$ where $X \subseteq_N U$. Then $\tau_R(X)$ satisfies the following axioms:

1) U and $\emptyset \in \tau_R(X)$

2) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,

3) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the **Nano topological space(NTS)**. The elements of $\tau_R(X)$ are called as **Nano open sets**. The complement of Nano-open sets is called **Nano closed sets**.

Definition 2.3 :[5] If $(U, \tau_R(X))$ is a NTS with respect to X and if $S \subseteq_N U$, then

• The **Nano interior** of S is defined as the union of all Nano open subsets of S and it is denoted by $Nint(S)$.That is, $Nint(S)$ is the largest open subset of S.

• The **Nano closure** of S is defines as the intersection of all Nano closed sets containing S it is denoted by $Ncl(S)$.That is, $Ncl(S)$is the smallest nano closed set containing S.

Definition 2.4 :[1] A **Nano generalized $\alpha*$ ($Ng\alpha^*$) closed** set is a subset S of a NTS $(U, \tau_R(X))$ if $N\alpha cl(S) \subseteq_N Nint^*(F)$whenever $S \subseteq_N F$and F is Ng-open in U.

Definition 2.5 : Let $(U, \tau_R(X))$ and $(V, \sigma_R(Y))$ be Nano topological spaces, then the mapping $f : (U, \tau_R(X)) \to (V, \sigma_R(Y))$ is called **Nano contra continuous function** if $f^{-1}(S)$ is nano closed in $(U, \tau_R(X))$ for every nano open set S in $(V, \sigma_R(Y))$.

Definition 2.6 :[2] A function $f : (U, \tau_R(X)) \to (V, \sigma_R(Y))$ is called **Nano generalized α^* ($Ng\alpha^*$) continuous function** if $f^{-1}(S)$ is $Ng\alpha^*$ closed in $(U, \tau_R(X))$ for every nano closed set S in $(V, \sigma_R(Y))$.

Definition 2.7 :[6] Let $(U, \tau_R(X))$ and $(V, \sigma_R(Y))$ be Nano topological spaces, then the mapping $f : (U, \tau_R(X)) \to (V, \sigma_R(Y))$ is called **Nano continuous function** if $f^{-1}(S)$ is nano open in $(U, \tau_R(X))$ for every nano open set S in $\left(V, \sigma_R(Y)\right)$.

Definition 2.8 :[2] A function $f : (U, \tau_R(X)) \to (V, \sigma_R(Y))$ is called **Nano generalized α^* ($Ng\alpha^*$) irresolute function** if $f^{-1}(S)$ is $Ng\alpha^*$ closed in $(U, \tau_R(X))$ for every $Ng\alpha^*$ closed set S in $(V, \sigma_R(Y))$.

Result 2.9:[1]

1. Every nano closed set is $Ng\alpha^*$ - closed.
2. Every nano open set is $Ng\alpha^*$ - open.
3. Every Ng^*-open set is $Ng\alpha^*$ - open.
4. Arbitrary union of $Ng\alpha^*$ open sets are $Ng\alpha^*$ open

Definition 2.10: A subset S of a NTS $(U, \tau_R(X))$ is called

1. Ng-closed[2] if $Ncl(S) \subseteq_N F$, whenever $S \subseteq_N F$ and F is Nano open in U.
2. Ng^*-closed[1] if $Ncl(S) \subseteq_N F$, whenever $S \subseteq_N F$ and F is Ng-open in U.

The complements of the above-mentioned sets are called their respective closed sets.

Definition 2.11 : Let $(U, \tau_R(X))$ and $(V, \sigma_R(Y))$ be Nano topological spaces, then the mapping $f : (U, \tau_R(X)) \to (V, \sigma_R(Y))$ is called **Ng^*-continuous** if $f^{-1}(S)$ is Ng^*-closed in $(U, \tau_R(X))$ for every nano closed set S in $\left(V, \sigma_R(Y)\right)$.

Definition 2.12 : For every set $A \subseteq U$, the **Nano generalized closure of A** is defined as the intersection of all Ng- closed sets containing A and is denoted by $Ncl^*(A)$.

Definition 2.13 : For every set $A \subseteq U$, the **Nano generalized interior of A** is defined as the union of all Ng- open sets contained in A and is denoted by $Nint^*(A)$.

III. CONTRA $Ng\alpha^*$ CONTINUOUS FUNCTIONS

Definition 3.1 : A function $f : (U, \tau_R(X)) \to (V, \sigma_R(Y))$ is called contra nano generalized α^* (contra $Ng\alpha^*$) continuous function if $f^{-1}(S)$ is $Ng\alpha^*$ closed in $(U, \tau_R(X))$ for every nano open set S in $(V, \sigma_R(Y))$.

Example 3.2 :

Let U = {p,q,r,s} with U/R = {{p},{r},{q,s}}. Let X = {p,q} $\subseteq_N U$.Then $\tau_R(X) = \{U, \emptyset,$ {p}, {q,s},{p,q,s}}.Here $Ng\alpha^*$ closed sets are { U, $\emptyset$,{r},{p,r},{q,r},{r,s},{p,q,r},{p,r,s},{q,r,s}}

Let V = {p,q,r,s} with V/R = {{r},{p,q,s}}. Let Y={q,s} $\subseteq_N V$.Then $\sigma_R(Y)$= {U, $\emptyset$, {p,q,s}}. Here $Ng\alpha^*$ closed sets = { U, $\emptyset$,{r},{p,r},{q,r},{r,s},{p,q,r},{p,r,s},{q,r,s}}. Let $f : (U,\tau_R(X)) \to (V,\sigma_R(Y))$ be defined by $f(p)$=q, $f(q) = p$, $f(r) = s$, $f(s)$= r. Then $f^{-1}(p) = q$, $f^{-1}(q) = p$, $f^{-1}(r) = s$, $f^{-1}(s) = r$. $f^{-1}(\{p,q,s\})$={p, q, r} $\in Ng\alpha^*$ closed in $(U,\tau_R(X))$ were {p,q,s} is nano open in $(V,\sigma_R(Y))$.Therefore f is contra $Ng\alpha*$ continuous function.

Theorem 3.3 : Every nano contra continuous function is contra $Ng\alpha^*$ continuous function.

Proof: Let $f : (U,\tau_R(X)) \to (V,\sigma_R(Y))$ be nano contra continuous function. Let S be nano open set in $(V,\sigma_R(Y))$. Since f is nano contra continuous function , $f^{-1}(S)$ nano closed in $(U,\tau_R(X))$.By Result 2.9(1), $f^{-1}(S)$ is $Ng\alpha^*$ closed in $(U,\tau_R(X))$.Therefore f is contra $Ng\alpha^*$ continuous function.

Remark 3.4 : The converse of the above theorem is not true. This is proved by following example.

Example 3.5 :

Let U = {a,b,c,d} with U/R = {{a},{c},{b,d}}. Let X = {a,b} $\subseteq_N U$.Then $\tau_R(X)$= {U, $\emptyset$,{a}, {b,d},{a,b,d}}. Nano closed sets are { U, $\emptyset$,{b,c,d},{a,c},{c}}. Here $Ng\alpha^*$closed sets are {U, $\emptyset$,{c},{a,c},{b,c},{c,d},{a,b,c},{a,c,d},{b,c,d}}. Let V = {a,b,c,d} with V/R = {{c},{a,b,d}}. Let Y={b,d} $\subseteq_N V$.Then $\sigma_R(Y)$= {U, $\emptyset$, {a,b,d}}. Here $Ng\alpha^*$ closed sets are { U, $\emptyset$,{c},{a,c},{b,c},{c,d},{a,b,c},{a,c,d},{b,c,d}}. Let $f : (U,\tau_R(X)) \to (V,\sigma_R(Y))$ be defined by $f(a)$=b, $f(b) = a$, $f(c) = d$, $f(d)$= c. Then $f^{-1}(a) = b$, $f^{-1}(b) = a$, $f^{-1}(c) = d$, $f^{-1}(d) = c$. $f^{-1}(\{a,b,d\})$={a, b, c} $\in Ng\alpha^*$ closed in $(U,\tau_R(X))$ were {a,b,d} is nano open in $(V,\sigma_R(Y))$.Therefore f is contra $Ng\alpha*$ continuous function.Since f^{-1}({a,b,d}) = {b,a,c} is not nano closed in $(U,\tau_R(X))$ were {a,b,d}is nano open in $(V,\sigma_R(Y))$, f is not nano contra continuous.

Theorem 3.6 : For a function $f : (U,\tau_R(X)) \to (V,\sigma_R(Y))$, the following are equivalent.

1) f is contra $Ng\alpha^*$ continuous.

2) For each $x \in U$ and each nano closed set F in $(V,\sigma_R(Y))$ containing $f(x)$,there exists a $Ng\alpha^*$ open set J in $(U,\tau_R(X))$ containing x such that $f(J) \subseteq F$

3) The inverse image of each nano closed set in $(V,\sigma_R(Y))$ is $Ng\alpha^*$ open in $(U,\tau_R(X))$.

Proof:

1) ⇒ 2)

Let $f:(U,\tau_R(X)) \to (V,\sigma_R(Y))$ be contra $Ng\alpha^*$ continuous. Let $x \in U$ and F be nano closed set in $(V,\sigma_R(Y))$ containing $f(x)$. Let $S = V/F$ be an nano open set in $(V,\sigma_R(Y))$ and $f(x) \notin V/F = S$. Since f is contra $Ng\alpha^*$ continuous, $f^{-1}(S)$ is $Ng\alpha^*$ closed in $(U,\tau_R(X))$ and $x \notin f^{-1}(S)$ then $f^{-1}(S) = f^{-1}(V/F) = U/f^{-1}(F)$ is $Ng\alpha^*$ closed in $(U,\tau_R(X))$ and $x \notin U/f^{-1}(F)$. Since $U/f^{-1}(F)$ is $Ng\alpha^*$ closed and $x \notin U/f^{-1}(F)$, $f^{-1}(F)$ is $Ng\alpha^*$ open and $x \in f^{-1}(F)$. Let $J = f^{-1}(F)$ be $Ng\alpha^*$ open in U and $x \in J$ such that $f(J) \subseteq F$

2) ⇒ 3)

Let F be a nano closed set in $(V,\sigma_R(Y))$.Let $x \in f^{-1}(F)$ then $f(x) \in F$.Then by our assumption there exists a $Ng\alpha^*$ open set U_x in $(U,\tau_R(X))$ containing x such that $f(x) \in f(U_x) \subseteq F$,.Then $f^{-1}(f(x)) \in f^{-1}(f(U_x)) \subseteq f^{-1}(F)$. So $x \in U_x \subseteq f^{-1}(F)$ Therefore $f^{-1}(F) = \cup \{U_x : x \in f^{-1}(F)\}$. Then by Result 2.9 (4), $f^{-1}(F)$ is $Ng\alpha^*$ open in $(U,\tau_R(X))$.

3) ⇒ 1)

Let F be nano open set in $(V,\sigma_R(Y))$ which implies V/F is nano closed in $(V,\sigma_R(Y))$.Then by our assumption , $f^{-1}(V/F) = U/f^{-1}(F)$ is $Ng\alpha^*$ open in $(U,\tau_R(X))$.Then $f^{-1}(F)$ is $Ng\alpha^*$ closed in $(U,\tau_R(X))$. Therefore f is contra $Ng\alpha^*$ continuous.

Theorem 3.7 : If $f:(U,\tau_R(X)) \to (V,\sigma_R(Y))$ is $Ng\alpha^*$ continuous and $g:(V,\sigma_R(Y)) \to (W,\gamma_R(Z))$ is nano contra continuous then $g \circ f:(U,\tau_R(X)) \to (W,\gamma_R(Z))$ is contra $Ng\alpha^*$ continuous.

Proof: Let S be a nano open set in $(W,\gamma_R(Z))$. Since g is nano contra continuous, $g^{-1}(S)$ is nano closed in $(V,\sigma_R(Y))$. Since f is $Ng\alpha^*$ continuous , $f^{-1}(g^{-1}(S)) = (g \circ f)^{-1}(S)$ is $Ng\alpha^*$ closed in $(U,\tau_R(X))$.Therefore $g \circ f$ is contra $Ng\alpha^*$ continuous.

Theorem 3.8 : If $f:(U,\tau_R(X)) \to (V,\sigma_R(Y))$ is contra $Ng\alpha^*$ continuous and $g:(V,\sigma_R(Y)) \to (W,\gamma_R(Z))$ is nano continuous then $g \circ f:(U,\tau_R(X)) \to (W,\gamma_R(Z))$ is contra $Ng\alpha^*$ continuous.

Proof: Let S be a nano open set in $(W, \gamma_R(Z))$. Since g is nano continuous ,$g^{-1}(S)$ is nano open in $(V, \sigma_R(Y))$.Since f is contra $Ng\alpha^*$ continuous,$f^{-1}(g^{-1}(S)) = (g \circ f)^{-1}(S)$ is $Ng\alpha^*$ closed in $(U, \tau_R(X))$.Therefore $g \circ f$ is contra $Ng\alpha^*$ continuous.

Theorem 3.9 : If $f : (U, \tau_R(X)) \to (V, \sigma_R(Y))$is contra $Ng\alpha^*$ continuous and $g : (V, \sigma_R(Y)) \to (W, \gamma_R(Z))$ is nano contra continuous then $g \circ f : (U, \tau_R(X)) \to (W, \gamma_R(Z))$ is $Ng\alpha^*$continuous.

Proof: Let S be a nano open set in $(W, \gamma_R(Z))$.Since g is nano contra continuous,$g^{-1}(S)$ is nano closed in $(V, \sigma_R(Y))$. Since f is contra $Ng\alpha^*$ continuous,,By Theorem 3.6, $f^{-1}(g^{-1}(S)) = (g \circ f)^{-1}(S)$ is $Ng\alpha^*$ open in $(U, \tau_R(X))$. Since $f : (U, \tau_R(X)) \to (V, \sigma_R(Y))$ is $Ng\alpha^*$ continuous iff the inverse image of every nano open set in $(V, \sigma_R(Y))$ is $Ng\alpha^*$ open in $(U, \tau_R(X))$, we have $g \circ f$ is $Ng\alpha^*$ continuous.

IV. CONTRA $Ng\alpha^*$ IRRESOLUTE FUNCTION

Definition 4.1 : A function $f : (U, \tau_R(X)) \to (V, \sigma_R(Y))$ is called contra nano generalized α^* (contra $Ng\alpha^*$) irresolute function if $f^{-1}(S)$ is $Ng\alpha^*$ closed in $(U, \tau_R(X))$ for every $Ng\alpha^*$ open set S in $(V, \sigma_R(Y))$.

Example 4.2 :

Let U = {p,q,r} with U/R = {{p},{q,r}}.Let X = {p} $\subseteq_N U$.Then $\tau_R(X) = \{U, \emptyset, \{p\}\}$

Here $Ng\alpha^*$ closed sets= { $U, \emptyset$,{r},{q},{q,r}}.Let V = {p,q,r} with V/R = {{p},{q},{r}}Let Y = {p,q} $\subseteq_N V$.Then $\sigma_R(Y) = \{U, \emptyset, \{p,q\}\}$.Here $Ng\alpha^*$ open sets = { $V, \emptyset$,{p},{q},{p,q}}.Let $f : (U, \tau_R(X)) \to (V, \sigma_R(Y))$ be defined by $f(p) = r, f(q) = p, f(r) = q$ then $f^{-1}(p) = q$,$f^{-1}(q) = r, f^{-1}(r) = p$. $f^{-1}(\{p,q\})$ = {q,r} , $f^{-1}(\{p\}) = q$, $f^{-1}(\{q\})$ = {r} then $f^{-1}(\{p,q\}), f^{-1}(\{p\}), f^{-1}(\{q\}) \in Ng\alpha^*$ closed in $(U, \tau_R(X))$ were {p,q},{p},{q} is $Ng\alpha^*$ open set in $(V, \sigma_R(Y))$.Since $f^{-1}(S)$ is $Ng\alpha^*$ closed in $(U, \tau_R(X))$ for every $Ng\alpha^*$ open set S in $(V, \sigma_R(Y))$, We have f is contra $Ng\alpha^*$ irresolute function.

Theorem 4.3: If $f : (U, \tau_R(X)) \to (V, \sigma_R(Y))$ is $Ng\alpha^*$ irresolute and $g : (V, \sigma_R(Y)) \to (W, \gamma_R(Z))$ is contra $Ng\alpha^*$ irresolute then $g \circ f : (U, \tau_R(X)) \to (W, \gamma_R(Z))$ is contra $Ng\alpha^*$ irresolute.

Proof: Let S be a $N g \alpha^{*}$ open set in $(W, \gamma_R(Z))$.Since g is contra $N g \alpha^{*}$ irresolute, $g^{-1}(S)$ is in $N g \alpha^{*}$ closed $(V, \sigma_R(Y))$. Since f is $N g \alpha^{*}$ irresolute,$f^{-1}(g^{-1}(S)) = (g \circ f)^{-1}(S)$ is $N g \alpha^{*}$ closed in $(U, \tau_R(X))$.Therefore $g \circ f$ is contra $N g \alpha^{*}$ irresolute.

Theorem 4.4 : For a function $f : (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$, the following are equivalent.

1) f is contra $N g \alpha^{*}$ irresolute.

2) For each $x \in U$ and each $N g \alpha^{*}$ closed set F in $(V, \sigma_R(Y))$ containing (x) , there exists a $N g \alpha^{*}$ open set J in $(U, \tau_R(X))$ containing x such that $f(J) \subseteq F$

3) The inverse image of each $N g \alpha^{*}$closed set in $(V, \sigma_R(Y))$ is $N g \alpha^{*}$ open in $(U, \tau_R(X))$.

Proof :

1) ⇒ 2)

Let $f : (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$ be contra $N g \alpha^{*}$ irresolute. Let $x \in U$ and F be $N g \alpha^{*}$ closed set in $(V, \sigma_R(Y))$ containing $f(x)$.Let $S = V/F$ be an $N g \alpha^{*}$ open set in $(V, \sigma_R(Y))$ and $f(x) \notin V/F = S$. Since f is contra $N g \alpha^{*}$ irresolute ,$f^{-1}(S)$ is $N g \alpha^{*}$ closed in $(U, \tau_R(X))$ and $x \notin f^{-1}(S)$. Then $f^{-1}(S) = f^{-1}(V/F)= U/f^{-1}(F)$ is $N g \alpha^{*}$ closed in $(U, \tau_R(X))$ and $x \notin U/f^{-1}(F)$. Since $U/f^{-1}(F)$ is $N g \alpha^{*}$ closed , $f^{-1}(F)$ is $N g \alpha^{*}$ open.Let $J = f^{-1}(F)$ be $N g \alpha^{*}$ open in $(U, \tau_R(X))$then $x \in J$ such that $f(J) \subseteq F$

2) ⇒ 3)

Let F be a $N g \alpha^{*}$closed set in $(V, \sigma_R(Y))$.Let $x \in f^{-1}(F)$ then $f(x) \in F$ .Then by our assumption there exists a $N g \alpha^{*}$ open set U_x in $(U, \tau_R(X))$ containing x such that $f(x) \in f(U_x) \subseteq F$. Then $f^{-1}(f(x)) \in f^{-1}(f(U_x)) \subseteq f^{-1}(F)$ which implies $x \in U_x \subseteq f^{-1}(F)$. Therefore $f^{-1}(F) = \cup \{U_x : x \in f^{-1}(F)\}$.Then by Result 2.9 (4), $f^{-1}(F)$ is $N g \alpha^{*}$ open in $(U, \tau_R(X))$.

3) ⇒ 1)

Let F be $N g \alpha^{*}$ open set in $(V, \sigma_R(Y))$.Since F is $N g \alpha^{*}$ open set in $(V, \sigma_R(Y))$,V/F is $N g \alpha^{*}$ closed in $(V, \sigma_R(Y))$ .Then by our assumption,$f^{-1}(V/F) = U/f^{-1}(F)$ is $N g \alpha^{*}$ open in $(U, \tau_R(X))$.Then $f^{-1}(F)$ is $N g \alpha^{*}$ closed in $(U, \tau_R(X))$. Therefore f is contra $N g \alpha^{*}$ irresolute.

Theorem 4.5 : If $f : (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$ is contra $N g \alpha^{*}$ irresolute and $g : (V, \sigma_R(Y)) \rightarrow (W, \gamma_R(Z))$ is contra $N g \alpha^{*}$ irresolute then $g \circ f : (U, \tau_R(X)) \rightarrow (W, \gamma_R(Z))$ is $N g \alpha^{*}$ irresolute.

Proof: Let S be a $Ng\alpha^*$ open set in $(W, \gamma_R(Z))$. Since g is contra $Ng\alpha^*$ irresolute,$g^{-1}(S)$ is in $Ng\alpha^*$ closed $(V, \sigma_R(Y))$.Since f is contra $Ng\alpha^*$ irresolute,By Theorem 4.4,$f^{-1}(g^{-1}(S)) = (g \circ f)^{-1}(S)$is $Ng\alpha^*$ open in $(U, \tau_R(X))$. Since f is $Ng\alpha^*$ irresolute iff the inverse image of each $Ng\alpha^*$ open set in $(V, \sigma_R(Y))$ is $Ng\alpha^*$ open in $(U, \tau_R(X))$, we have $g \circ f$ is $Ng\alpha^*$ irresolute.

Theorem 4.6 : Every contra $Ng\alpha^*$ irresolute function is contra $Ng\alpha^*$ continuous function.

Proof: Let $f : (U, \tau_R(X)) \to (V, \sigma_R(Y))$ be contra $Ng\alpha^*$ irresolute function.Let S be nano open set in $(V, \sigma_R(Y))$. By Result 2.9(2),we have S is $Ng\alpha^*$ open set in$(V, \sigma_R(Y))$ Since f is contra $Ng\alpha^*$ irresolute function,$f^{-1}(S)$ $Ng\alpha^*$ closed in $(U, \tau_R(X))$.Therefore f is contra $Ng\alpha^*$ continuous function.

Remark 4.7 : The converse of the above theorem is not true. This is proved by following example.

Example 4.8 :

Let U = {a,b,c,d} with U/R = {{a},{c},{b,d}}. Let X = {a,b} $\subseteq_N U$.Then $\tau_R(X)$ = {U, ∅,{a}, {b,d},{a,b,d}}.Here $Ng\alpha^*$closedsets = { U, ∅,{c},{a,c},{b,c},{c,d},{a,b,c},{a,c,d},{b,c,d}}

Let V = {a,b,c,d} with V/R = {{c},{a,b,d}}. Let Y={b,d} $\subseteq_N V$.Then $\sigma_R(Y)$= {U, ∅, {a,b,d}}. $Ng\alpha^*$ open sets = { U, ∅,{a,b,d},{b,d},{a,d},{a,b},{d},{b},{a}}. Here $Ng\alpha^*$ closed sets = { U, ∅,{c},{a,c},{b,c},{c,d},{a,b,c},{a,c,d},{b,c,d}}. Let $f : (U, \tau_R(X)) \to (V, \sigma_R(Y))$ be defined by $f(a)$=b, $f(b) = a$, $f(c) = d$, $f(d)$= c. Then $f^{-1}(a) = b$, $f^{-1}(b) = a$, $f^{-1}(c) = d$, $f^{-1}(d) = c$. $f^{-1}(\{a, b, d\})$={a, b, c} ∈ $Ng\alpha^*$ closed in $(U, \tau_R(X))$ were {a,b,d} is nano open in $(V, \sigma_R(Y))$.Therefore f is contra $Ng\alpha*$ continuous function. Since f^{-1}({a,b}) = {b,a} is not $Ng\alpha^*$closed in $(U, \tau_R(X))$ were {a,b}is $Ng\alpha^*$ open in $(V, \sigma_R(Y))$, f is not contra $Ng\alpha^*$ irresolute function.

Theorem 4.9 : Let $f : (U, \tau_R(X)) \to (V, \sigma_R(Y))$ and $g : (V, \sigma_R(Y)) \to (W, \gamma_R(Z))$ be two functions

1. If f is contra $Ng\alpha^*$ irresolute and g is $Ng\alpha^*$ irresolute then $g \circ f$ is contra $Ng\alpha^*$ irresolute.
2. If f is contra $Ng\alpha^*$ irresolute. and g is $Ng\alpha^*$continuous then $g \circ f$ is contra $Ng\alpha^*$ continuous.

3. If f is contra $Ng\alpha^*$ irresolute and g is nano continuous then $g \circ f$ is contra $Ng\alpha^*$ continuous.
4. If f is contra $Ng\alpha^*$ irresolute and g is Ng^* continuous then $g \circ f$is contra $Ng\alpha^*$ continuous.

Proof :

1) Let S be a $Ng\alpha^*$ open set in $(W, \gamma_R(Z))$. Since g is $Ng\alpha^*$ irresolute , $g^{-1}(S)$ is $Ng\alpha^*$ open in $(V, \sigma_R(Y))$. Since f is contra $Ng\alpha^*$ irresolute,$f^{-1}(g^{-1}(S)) = (g \circ f)^{-1}(S)$ is $Ng\alpha^*$ closed in $(U, \tau_R(X))$.Therefore $g \circ f$ is contra $Ng\alpha^*$ irresolute.

2) Let S be a nano open set in $(W, \gamma_R(Z))$. Since g is $Ng\alpha^*$ continuous, $g^{-1}(S)$ is $Ng\alpha^*$ open in $(V, \sigma_R(Y))$. Since f is contra $Ng\alpha^*$ irresolute, $f^{-1}(g^{-1}(S)) = (g \circ f)^{-1}(S)$ is $Ng\alpha^*$ closed in $(U, \tau_R(X))$.Therefore $g \circ f$ is contra $Ng\alpha^*$ continuous.

3) Let S be a nano open set in $(W, \gamma_R(Z))$. Since g is nano continuous, $g^{-1}(S)$ is nano open in $(V, \sigma_R(Y))$. By Result 2.9(2), we have $g^{-1}(S)$ is $Ng\alpha^*$ open $(V, \sigma_R(Y))$.Since f is contra $Ng\alpha^*$ irresolute,$f^{-1}(g^{-1}(S)) = (g \circ f)^{-1}(S)$ is $Ng\alpha^*$ closed in $(U, \tau_R(X))$.Therefore $g \circ f$ is contra $Ng\alpha^*$ continuous.

4) Let S be a nano open set in $(W, \gamma_R(Z))$. Since g is Ng^* continuous, $g^{-1}(S)$ is Ng^* open in $(V, \sigma_R(Y))$. By Result 2.9(3), we have $g^{-1}(S)$ is $Ng\alpha^*$ open in $(V, \sigma_R(Y))$.Since f is contra $Ng\alpha^*$ irresolute,$f^{-1}(g^{-1}(S)) = (g \circ f)^{-1}(S)$ is $Ng\alpha^*$ closed in $(U, \tau_R(X))$.Therefore $g \circ f$ is contra $Ng\alpha^*$ continuous.

V. CONCLUSION

In this paper, we have introduced Contra Nano $g\alpha^*$ continuous functions and Contra Nano $g\alpha^*$- irresolute functions and explored their fundamental properties. We have examined their relationships with various existing Nano continuous and Nano irresolute functions. Furthermore, several compositions of these functions have been investigated, providing deeper insights into their structural behaviour. These findings contribute to the ongoing study of function spaces in Nano topological settings.

REFERENCES

[1] Anbarasi Rodrigo P, Subithra P. On A New Class of Nano Generalized Closed Sets in Nano Topological Spaces, presented in International Conference On Advances In Science And Engineering (ICASE-2022), (communicated).

[2] Anbarasi Rodrigo P, Subithra P. Nano $g\alpha^*$-continuous functions in Nano topological spaces, presented in Journal of Research in Applied Mathematics Volume 8 – Issue 11 (2022) p: 01-07.

[3] Bhuvaneshwari K, Mythili Gnanapriya K. On Nano Generalized closed sets, International Journal of Scientific and Research Publications, 2014, 4(5): p.1-3.

[4] Bhuvaneshwari K, Ezhilarasi K. On Nano semi-generalized and Nano generalizedsemi- closed sets, International Journal of Mathematics and Computer Applications Research, 2014, 4(3): p. 117-124.

[5] LellisThivagar M, Carmel Richard. On Nano forms of weakly open sets, International Journal of Mathematical and Statistics Invention.,2012,1(1):p.31- 37.

[6] LellisThivagar M and Carmel Richard. On Nano continuity, Mathematical Theory Model, 2013, 3(7): p. 32-37.