ON MICRO α^*-HOMEOMORPHISM IN MICRO TOPOLOGICAL SPACES

A.Anisha Briskillal

Department of Mathematics, St. Mary's College (Autonomous) Thoothukudi

Affiliated to Manonmaniam Sundaranar University, Tirunelveli, Tamilnadu, India.

Abstract: The aim of this paper is to introduce and study the concept of Micro α^*-homeomorphism in Micro topological spaces.

Keywords: Micro α^*-open, Micro α^*-continuous, Micro α^*-open map and Micro α^*-homeomorphism.

I. INTRODUCTION

Nano Topology was introduced by Lellis Thivagar[6] in the year 2013. This topology is based on the concept of lower approximation, upper approximation and boundary region. It has maximum five open sets and minimum three open sets including universal & empty set. Every Nano Topology is Micro Topology. Micro Topology was introduced by Sakkraiveeranan Chandrasekar[3] in 2019. Micro topology was the extension of Nano Topology. In this paper, we introduced Micro α*-homeomorphism in Micro Topological Space.

II. PRELIMINARIES

Definition 2.1 [3]:

Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

1) **The lower approximation of X** with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is

$L_R(X) = \bigcup_{x \in U}\{R(x): R(x) \subseteq X\}$ where $R(x)$ denotes the equivalence class determined by $x \in U$.

2) **The upper approximation of X** with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$**.** That is

$U_R(X) = \bigcup_{x \in U}\{R(x): R(x) \cap X \neq \emptyset\}$

3) The **boundary region of X** with respect to R is the set of all objects, which can be classified neither as X nor as not $-X$ with respect to R and it is denoted by $B_R(X)$. That is

$B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2 [3]:

Let U be an universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \emptyset, U_R(X), L_R(X), B_R(X)\}$ where $X \subseteq U$ satisfies the following axioms:

1) U and $\emptyset \in \tau_R(X)$
2) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,
3) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the **nano topological space(NTS)**. The elements are called as **nano open Sets**. The complement of nano-open sets is called **nano closed sets**.

Definition 2.3 [3]:

$(U, \tau_R(X))$ is a nano topological space here $\mu_R(X) = \{N \cup (N' \cap \mu)\}/N, N' \in \tau_R(X)$ and it is called micro topology of $\tau_R(X)$ by μ where $\mu \notin \tau_R(X)$. The micro topology $\mu_R(X)$ satisfies the following axioms:

1) U and $\emptyset \in \mu_R(X)$
2) The union of the elements of any sub collection of $\mu_R(X)$ is in $\mu_R(X)$,
3) The intersection of the elements of any finite sub collection of $\mu_R(X)$ is in $\mu_R(X)$.

Then $\mu_R(X)$ is called the micro topology on U with respect to X. The triplet $(U, \tau_R(X), \mu_R(X))$ is called **micro topological spaces** and the elements of $\mu_R(X)$ are called **micro open set** and the complement of micro open set is called a **micro closed set.**

Definition 2.4 [3]:

The **micro closure** of a set A is denoted by $M\text{-}cl(A)$ and is defined as $M\text{-}cl(A) = \cap\{B : B \text{ } is\ micro\ closed\ and\ A \subseteq B\}$.

Definition 2.5 [3]:

The **micro interior** of a set A is denoted by $M\text{-}int(A)$ and is defined as $M\text{-}int(A) = \cup\{B : B\ is\ micro\ open\ and\ A \supseteq B\}$.

Definition 2.6 [6]:

Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space. A subset A of U is said to be **micro generalized closed set** if $Mcl(A) \subseteq U$, Whenever $A \subseteq U$ and U is micro open in U. The complement of micro generalized Closed set is **micro generalized open set.**

Definition 2.7 [4]:

Let $(U,\tau_R(X),\mu_R(X))$ be a micro topological space. A set A is called an **$M\alpha$-open set** if $A \subseteq M\text{-}int(M\text{-}cl(M\text{-}int(A)))$.

Let $(U,\tau_R(X),\mu_R(X))$ be a micro topological space. Then the complement of a $M\alpha$-open set is called **$M\alpha$-closed set**.

Definition 2.8 [2]:

Let $(U,\tau_R(X),\mu_R(X))$ be a micro topological space. A set A is called an **$M\alpha g$-open set** if $M\text{-}\alpha cl\ (A) \subseteq U$ whenever $A \subseteq U$ and U is micro-open in U.

Let $(U,\tau_R(X),\mu_R(X))$ be a micro topological space. Then the complement of a $M\alpha g$-open set is called **$M\alpha g$-closed set**.

Definition 2.9 [1]:

Let $(U,\tau_R(X),\mu_R(X))$ be a micro topological space. A set A is called an **$M\alpha^*$-open set** if $A \subseteq M\text{-}int^*(M\text{-}cl(M\text{-}int^*(A)))$.

Let $(U,\tau_R(X),\mu_R(X))$ be a micro topological space. Then the complement of a $M\alpha^*$-open set is called **$M\alpha^*$-closed set**.

Result 2.10 [1]:

(i) Every micro-open set is micro α^*- open.

(ii) Every micro α-open set is micro α^*- open.

(iii)Every micro g-open set is micro α^*- open.

(iv)Every micro αg-open set is micro α^*- open.

Definition 2.11 [5]:

A bijective function $f:(U,\tau_R(X),\mu_R(X)) \to (V,\tau'_R(Y),\mu'_R(Y))$ is called **micro homeomorphism** if f is both micro continuous and micro open map.

Definition 2.12:

A function $f:(U,\tau_R(X),\mu_R(X)) \to (V,\tau'_R(Y),\mu'_R(Y))$ is called **micro α^*-continuous** if $f^{-1}(S)$ is micro α^*-open in $(U,\tau_R(X),\mu_R(X))$ for every micro open set S in $(V,\tau'_R(Y),\mu'_R(Y))$.

Definition 2.13:

A function $f:(U,\tau_R(X),\mu_R(X)) \to (V,\tau'_R(Y),\mu'_R(Y))$ is called **micro α^*-open map** if for any micro open set S in $(U,\tau_R(X),\mu_R(X))$, the image $f(S)$ is micro α^*-open in $(V,\tau'_R(Y),\mu'_R(Y))$. Then the complement of a $M\alpha^*$-open map is called **micro α^*-closed map**.

Definition 2.14:

A bijective function $f : (U,\tau_R(X),\mu_R(X)) \to (V,\tau'_R(Y),\mu'_R(Y))$ is called **micro α-homeomorphism** if f is both micro α-continuous and micro α-open map.

Definition 2.15 [5]:

A bijective function $f : (U,\tau_R(X),\mu_R(X)) \to (V,\tau'_R(Y),\mu'_R(Y))$ is called **micro αg-homeomorphism** if f is both micro αg-continuous and micro αg-open map.

Definition 2.16:

A bijective function $f : (U,\tau_R(X),\mu_R(X)) \to (V,\tau'_R(Y),\mu'_R(Y))$ is called **micro g-homeomorphism** if f is both micro g-continuous and micro g-open map.

III. MICRO α^*-HOMEOMORPHISM

In this section we introduce the concept of Micro α^*-homeomorphism and some of their properties are discussed below.

Definition3.1:

A bijective function $f : (U,\tau_R(X),\mu_R(X)) \to (V,\tau'_R(Y),\mu'_R(Y))$ is called **micro α^*-homeomorphism** if f is both micro α^*-continuous and micro α^*-open map.

Example 3.2:

Let $U = \{a,b,c,d\}$ with $U/R = \{\{a,b\},\{c,d\}\}$ and $X = \{a,b\} \subseteq U$. Then $\tau_R(X) = \{U,\phi,\{a,b\}\}$ and then $\mu = \{a,b,c\} \notin \tau_R(X)$, $\mu_R(X) = \{U, \emptyset, \{a,b\}, \{a,b,c\}\}$. Micro α^*-open sets are $\{U,\phi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\}$, $\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}$. Let $V = \{a,b,c,d\}$ with $V/R = \{a,b,\{c,d\}\}$ and $Y = \{b,d\} \subseteq V$. Then $\tau'_R(Y) = \{U,\phi,\{b\},\{c,d\},\{b,c,d\}\}$ and then $\mu' = \{a,b\} \notin \tau'_R(Y)$, $\mu'_R(Y) = \{V,\phi,\{b\},\{c,d\},\{b,c,d\},\{a,b\}\}$, Micro α^*-open sets are $\{V,\emptyset,\{b\},\{c\},\{d\},\{a,b\},\{b,c\},\{b,d\},\{c,d\},\{a,b,c\},\{a,b,d\},\{b,c,d\}\}$.

Let $f : (U,\tau_R(X),\mu_R(X)) \to (V,\tau'_R(Y),\mu'_R(Y))$ be a function defined by $f(a) = d, f(b) = b, f(c) = c, f(d) = a$. Then $f^{-1}(a) = d, f^{-1}(b) = b, f^{-1}(c) = c, f^{-1}(d) = a$. Since $f^{-1}(\{b\}) = \{b\}$, $f^{-1}(\{c,d\}) = \{a,c\}$, $f^{-1}(\{b,c,d\}) = \{a,b,c\}$, $f^{-1}(\{a,b\}) = \{b,d\}$ are micro α^*-open in $(U,\tau_R(X),\mu_R(X))$ were $\{b\},\{c,d\},\{b,c,d\},\{a,b\}$ are Micro open in $(V,\tau'_R(Y),\mu'_R(Y))$. Therefore f is micro α^*-continuous. Since $f(\{a,b\}) = \{b,d\}$ and $f(\{a,b,c\}) = \{b,c,d\}$ are Micro α^*-open in $(V,\tau'_R(Y),\mu'_R(Y))$ were $\{a,b\}$ and $\{a,b,c\}$ are Micro open in $(U,\tau_R(X),\mu_R(X))$. Hence f is micro α^*-open map. Therefore f is micro α^*-homeomorphism.

Theorem3.3:

Every micro homeomorphism is a micro α^*-homeomorphism.

Proof:

Let f be micro homeomorphism. Then f is micro continuous and micro open map. Let S be micro open in $(V, \tau'_R(Y), \mu'_R(Y))$. Since f is micro continuous, $f^{-1}(S)$ is micro open in $(U, \tau_R(X), \mu_R(X))$. Since every micro open set is micro α^* open. $f^{-1}(S)$ is micro α^*-open in $(U, \tau_R(X), \mu_R(X))$. Therefore f is micro α^*-continuous. Let A be micro open in $(U, \tau_R(X), \mu_R(X))$. Since f is micro open map, $f(S)$ is micro open in $(V, \tau'_R(Y), \mu'_R(Y))$. Therefore $f(S)$ is micro α^*-open in $(V, \tau'_R(Y), \mu'_R(Y))$. Therefore f is micro α^*-open map. Since f is micro α^*-continuous and micro α^*-open map, f is micro α^*-homeomorphism.

Remark3.4:

The converse of the above theorem need not to be true. This is proved by the following example.

Example3.5:

Let $U = \{p, q, r, s\}$ with $U/R = \{\{p, q\}, \{r, s\}\}$ and $X = \{p, q\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{p, q\}\}$ and then $\mu = \{p, q, r\} \notin \tau_R(X)$, $\mu_R(X) = \{U, \emptyset, \{p, q\}, \{p, q, r\}\}$, Micro α^*-open sets are $\{U, \emptyset, \{p\}, \{q\}, \{r\}, \{p, q\}, \{p, r\}, \{p, s\}, \{q, r\}, \{q, s\}, \{p, q, r\},$ $\{p, q, s\}, \{p, r, s\}, \{q, r, s\}\}$. Let $V = \{p, q, r, s\}$ with $V/R = \{p, q, \{r, s\}\}$ and $Y = \{q, s\} \subseteq V$. Then $\tau'_R(Y) = \{V, \phi, \{q\}, \{r, s\}, \{q, r, s\}\}$ and then $\mu' = \{p, q\} \notin \tau'_R(Y)$, $\mu'_R(Y) = \{V, \emptyset, \{q\}, \{r, s\}, \{q, r, s\}, \{p, q\}\}$, Micro α^*-open sets are $\{V, \emptyset, \{q\}, \{r\}, \{s\}, \{p, q\}, \{q, r\}$ $\{q, s\}, \{r, s\}, \{p, q, r\}, \{p, q, s\}, \{q, r, s\}\}$. Let $f : (U, \tau_R(X), \mu_R(X)) \to (V, \tau'_R(Y), \mu'_R(Y))$ be a function defined by $f(p) = s, f(q) = q, f(r) = r, f(s) = p$. Then $f^{-1}(p) = s, f^{-1}(q) = q, f^{-1}(r) = r, f^{-1}(s) = p$. Clearly f is micro α^*-homeomorphism. But $f^{-1}(\{q\}) = \{q\}$, $f^{-1}(\{r, s\}) = \{r, p\}$ are not micro open in $(U, \tau_R(X), \mu_R(X))$ were {q} and $\{r, s\}$ are micro open in $(V, \tau'_R(Y), \mu'_R(Y))$, f is not micro continuous. Since $f(\{p, q\}) = \{s, q\}$ is not micro α^*-open in $(V, \tau'_R(Y), \mu'_R(Y))$ were $\{p, q\}$ is micro open in $(U, \tau_R(X), \mu_R(X))$, f is not micro open map. Therefore f is not micro homeomorphism.

Theorem 3.6:

Every micro α-homeomorphism is micro α^*-homeomorphism.

Proof:

Let $f : (U, \tau_R(X), \mu_R(X)) \to (V, \tau'_R(Y), \mu'_R(Y))$ be an micro α-homeomorphism, then f is bijective, micro α-continuous and micro α-open map. Let V be an micro open set in $(V, \tau'_R(Y), \mu'_R(Y))$. Since f is micro α-continuous, $f^{-1}(V)$ is micro α-open in $(U, \tau_R(X), \mu_R(X))$. Since every micro α-open set is micro α^*-open, $f^{-1}(V)$ is micro α^*-open in $(U, \tau_R(X), \mu_R(X))$ which implies f is micro α^*-continuous. Let S be an micro open set in $(U, \tau_R(X), \mu_R(X))$. Since f is micro α-open map, $f(S)$ is micro α-open in $(V, \tau'_R(Y), \mu'_R(Y))$. Since every micro α-open set is micro α^*-open, $f(S)$ is micro α^*-open in $(V, \tau'_R(Y), \mu'_R(Y))$ which implies f is micro α^*-open map. Thus f is microα^*-homeomorphism.

Remark 3.7:

The converse of the above theorem need not be true. This is proved by the following example.

Example 3.8:

Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c\}, \{d\}\}$ and $X = \{b, d\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{b, d\}\}$ and then $\mu = \{c\} \notin \tau_R(X)$, $\mu_R(X) = \{U, \phi, \{a\}, \{b, d\}, \{a, b, d\}\}$, Micro α^*-open sets are $\{U, \phi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}$. Then Micro α-open sets are $\{U, \phi, \{a\}, \{b, d\}, \{a, b, d\}$. Let $V = \{a, b, c, d\}$ with $V/R = \{\{a\}, \{d\}\}$ and $Y = \{\{a\}, \{d\}\} \subseteq V$. Then $\tau'_R(Y) = \{V, \phi, \{a, b\}, \{d\}, \{a, b, d\}\}$ and $\mu' = \{c\} \notin \tau'_R(Y)$, $\mu'_R(Y) = \{V, \phi, \{c\}, \{d\}, \{c, d\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$, Micro α^*-open sets are $\{V, \phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau'_R(Y), \mu'_R(Y))$ be a function defined by $f(a) = d, f(b) = c, f(c) = b, f(d) = a$. Then $f^{-1}(a) = d, f^{-1}(b) = c, f^{-1}(c) = b, f^{-1}(d) = a$. Since $f^{-1}(\{c\}) = \{b\}$, $f^{-1}(\{d\}) = \{a\}, f^{-1}(\{c, d\}) = \{a, b\}, f^{-1}(\{a, b\}) = \{c, d\}, f^{-1}(\{a, b, c\}) = \{b, c, d\}, f^{-1}(\{a, b, d\}) = \{a, c, d\}$ are micro α^*-open in $(U, \tau_R(X), \mu_R(X))$ were $\{c\}, \{d\}, \{c, d\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}$ are micro-open in $(V, \tau'_R(Y), \mu'_R(Y))$. Thus f is micro α^*-continuous. Since $f(\{a\}) = \{d\}, f(\{b, d\}) = \{a, c\}, f(\{a, b, d\}) = \{a, c, d\}$ are micro α^*-open in $(V, \tau'_R(Y),$

$\mu'_R(Y))$, were $\{a\},\{b,d\},\{a,b,d\}$ are micro-open in $(U,\tau_R(X),\ \mu_R(X))$. Hence f is micro α^*-open map. Therefore f is micro α^*-homeomorphism. Since $f^{-1}(\{c,d\} = \{a,b\}$ is not micro α-open in $(U,\tau_R(X),\ \mu_R(X))$, were $\{c,d\}$ is Micro-open in $(V,\ \tau'_R(Y),\ \mu'_R(Y))$. Therefore f is not micro α-continuous and hence f is not micro α-homeomorphism.

Theorem 3.9:

Every micro g-homeomorphism is micro α^*-homeomorphism

Proof:

Let $f:(U,\tau_R(X),\mu_R(X)) \to (V,\tau'_R(Y),\mu'_R(Y))$ be an micro g-homeomorphism, then f is bijective, micro g-continuous and micro g-open map. Let V be an micro open set in $(V,\tau'_R(Y),\mu'_R(Y))$. Since f is micro g-continuous, $f^{-1}(V)$ is micro g-open in $(U,\tau_R(X),\mu_R(X))$. Since every micro g-open set is micro α^*-open, $f^{-1}(V)$ is micro α^*-open in $(U,\tau_R(X),\mu_R(X))$ which implies f is micro α^*-continuous. Let S be an micro open set in $(U,\tau_R(X),\mu_R(X))$. Since f is micro g-open map, $f(S)$ is micro g-open in $(V,\tau'_R(Y),\mu'_R(Y))$. Since every micro g-open set is micro α^*-open, $f(S)$ is micro α^*-open in $(V,\tau'_R(Y),\mu'_R(Y))$ which implies f is micro α^*-open map. Thus f is micro α^*-homeomorphism.

Remark 3.10:

The converse of the above theorem need not be true. This is proved by the following example.

Example 3.11:

Let $U = \{p,q,r,s\}$ with $U/R = \{\{p\},\{q\},\{r\},\{s\}\}$ and $X = \{q,s\} \subseteq U$. Then $\tau_R(X) = \{U,\phi,\{q,s\}\}$ and then $\mu = \{r\} \notin \tau_R(X)$, $\mu_R(X) = \{U,\ \phi,\ \{p\},\ \{q,s\},\ \{p,q,s\}\}$, Micro α^*-open sets are $\{U,\phi,\{p\},\{q\},\ \{s\},\ \{p,q\},\ \{p,s\},\ \{q,s\},\ \{q,r\},\ \{r,s\},\ \{p,q,r\},$ $\{p,q,s\},\{p,r,s\},\{q,r,s\}.Then$ Micro g-open sets are$\{U,\phi,\{p\},\{q\},\{s\},\{p,q\},\{p,s\},$ $\{q,s\},\{p,q,s\}\}$. Let $V = \{p,q,r,s\}$ with $V/R = \{\{p\},\{s\}\}$ and $Y = \{\{p\},\{s\}\} \subseteq V$. Then $\tau'_R(Y) = \{V,\phi,\{p,q\},\{s\},\{p,q,s\}\}\ and\ \mu' = \{r\} \notin \tau'_R(Y)$, $\mu'_R(Y) = \{V,\ \emptyset,\ \{r\},\ \{s\},$ $\{r,s\},\{p,q\},\{p,q,r\},\{p,q,s\}\}$, Micro α^*-open sets are $\{V,\phi,\{p\},\{q\},\{r\},\{s\},\{p,q\},$ $\{p,r\},\{p,s\},\{q,r\},\{q,s\},\{r,s\},\{p,q,r\},\{p,q,s\},\{p,r,s\},\{q,r,s\}\}$. Let $f:(U,\tau_R(X),$ $\mu_R(X)) \to (V,\ \tau'_R(Y),\ \mu'_R(Y))$ be a function defined by $f(p) = s, f(q) = r, f(r) = q, f(s) = p$. Then

$f^{-1}(p) = s, f^{-1}(q) = r, f^{-1}(r) = q, f^{-1}(s) = p$. Since

$f^{-1}(\{r\}) = \{q\}$,

$f^{-1}(\{s\}) = \{p\}$, $f^{-1}(\{r,s\}) = \{p,q\}$, $f^{-1}(\{p,q\}) = \{r,s\}$, $f^{-1}(\{p,q,r\}) = \{q,r,s\}, f^{-1}(\{p,q,s\}) = \{p,r,s\}$ are micro α^*-open in $(U, \tau_R(X), \mu_R(X))$ were $\{r\},\{s\},\{r,s\},\{p,q\},\{p,q,r\},\{p,q,s\}$ are micro-open in $(V, \tau'_R(Y), \mu'_R(Y))$. Thus f is micro α^*-continuous. Since $f(\{p\}) = \{s\}, f(\{q,s\}) = \{p,r\}, f(\{p,q,s\}) = \{p,r,s\}$ are micro α^*-open in $(V, \tau'_R(Y), \mu'_R(Y))$, were $\{p\},\{q,s\},\{p,q,s\}$ are micro-open in $(U, \tau_R(X), \mu_R(X))$. Hence f is micro α^*-open map. Therefore f is micro α^*-homeomorphism. Since $f^{-1}(\{p,q\}) = \{r,s\}$ is not micro g-open in $(U, \tau_R(X), \mu_R(X))$, were $\{p,q\}$ is micro-open in $(V, \tau'_R(Y), \mu'_R(Y))$. Therefore f is not micro g-continuous and hence f is not micro g-homeomorphism.

Theorem 3.12:

Every micro αg-homeomorphism is micro α^*-homeomorphism.

Proof:

Let $f : (U, \tau_R(X), \mu_R(X)) \to (V, \tau'_R(Y), \mu'_R(Y))$ be an micro αg-homeomorphism, then f is bijective, micro αg-continuous and micro αg-open map. Let V be an micro open set in $(V, \tau'_R(Y), \mu'_R(Y))$. Since f is micro αg-continuous, $f^{-1}(V)$ is micro αg-open in $(U, \tau_R(X), \mu_R(X))$. Since every micro αg-open set is micro α^*-open, $f^{-1}(V)$ is micro α^*-open in $(U, \tau_R(X), \mu_R(X))$ which implies f is micro α^*-continuous. Let S be an micro open set in $(U, \tau_R(X), \mu_R(X))$. Since f is micro αg-open map, $f(S)$ is micro αg-open in $(V, \tau'_R(Y), \mu'_R(Y))$. Since every micro αg-open set is micro α^*-open, $f(S)$ is micro α^*-open in $(V, \tau'_R(Y), \mu'_R(Y))$ which implies f is micro α^*-open map. Thus f is micro α^*-homeomorphism.

Remark 3.13:

The converse of the above theorem need not be true. This is proved by the following example.

Example 3.14:

Let $U = \{a,b,c,d\}$ with $U/R = \{\{a,b\},\{c,d\}\}$ and $X = \{a,b\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{a,b\}\}$ and then $\mu = \{a,b,c\} \notin \tau_R(X)$, $\mu_R(X) = \{U, \phi, \{a,b\}, \{a,b,c\}\}$, $Micro\ \alpha^*$ - $open\ sets\ are \{U, \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,$ d},$\{b,c,d\}\}$. Then Micro αg- open sets are$\{U, \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$.

Let $V = \{a, b, c, d\}$ with $V/R = \{\{a, b\}, \{c\}\}$ and $Y = \{b, c\} \subseteq V$. Then $\tau'_R(Y) = \{V, \phi, \{c\}, \{a, b\}, \{a, b, c\}\}$ and $\mu' = \{a\} \notin \tau'_R(Y)$, $\mu'_R(Y) = \{V, \emptyset, \{a\}, \{c\}, \{a, c\}, \{a, b\}, \{a, b, c\}\}$, Micro α^* - open sets are $\{V, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, c, d\}\}$. Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau'_R(Y), \mu'_R(Y))$ be a function defined by $f(a) = c$, $f(b) = a, f(c) = d, f(d) = b$. Then $f^{-1}(a) = b, f^{-1}(b) = d, f^{-1}(c) = a, f^{-1}(d) = c$. Since $f^{-1}(\{a\}) = \{b\}$, $f^{-1}(\{c\}) = \{a\}, f^{-1}(\{a, c\}) = \{a, b\}, f^{-1}(\{a, b\}) = \{b, d\}, f^{-1}(\{a, b, c\}) = \{a, b, d\}$ are micro α^*-open in $(U, \tau_R(X), \mu_R(X))$ were $\{a\}, \{c\}, \{a, c\}, \{a, b\}, \{a, b, c\}$ are micro-open in $(V, \tau'_R(Y), \mu'_R(Y))$. Thus f is micro α^*-continuous. Since $f(\{a, b\}) = \{a, c\}, f(\{a, b, c\}) = \{a, c, d\}$ are micro α^*-open in $(V, \tau'_R(Y), \mu'_R(Y))$ were $\{a, b\}, \{a, b, c\}$ are micro-open in $(U, \tau_R(X), \mu_R(X))$. Hence f is Micro α^*-open map. Therefore f is Micro α^*-homeomorphism. Since $f^{-1}(\{a, b\}) = \{b, d\}$ is not micro αg-open in $(U, \tau_R(X), \mu_R(X))$ were $\{a, b\}$ is micro-open in $(V, \tau'_R(Y), \mu'(Y))$. Therefore f is not micro αg-open map and hence f is not micro αg-homeomorphism.

Remark 3.15:

The composition of two micro α^*-homeomorphism need not be a micro α^*-homeomorphism.

Example 3.16:

Let $U = \{a, b, c, d)$ with $U/R = \{\{a\}, \{b\}, \{c\}, \{d\}\}$ and $X = \{b, d\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{b, d\}\}$ and then $\mu = \{c\} \notin \tau_R(X), \mu_R(X) = \{U, \phi, \{a\}, \{b, d\}, \{a, b, d\}\}$, Micro α^*-open sets are $\{U, \phi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}$. Let $V = \{a, b, c, d\}$ with $V/R = \{\{a\}, \{d\}\}$ and $Y = \{\{a\}, \{d\}\} \subseteq V$. Then $\tau'_R(Y) = \{V, \phi, \{a, b\}, \{d\}, \{a, b, d\}\}$ and $\mu' = \{c\} \notin \tau'_R(Y)$, $\mu'_R(Y) = \{V, \phi, \{c\}, \{d\}, \{c, d\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$, Micro α^*-open sets are $\{V, \phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. Let $W = \{a, b, c, d\}$ with $W/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $Z = \{c\} \subseteq W$. Then $\tau''_R(Z) = \{W, \phi, \{a, c\}\}$ and $\mu'' = \{a, d\} \notin \tau''_R(Z), \mu''_R(Z) = \{W, \emptyset, \{a\}, \{a, d\}, \{a, c\}, \{a, c, d\}\}$, Micro α^*-open sets are $\{W, \phi, \{a\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{c, d\}, \{a, b, c\},$

$\{a,c,d\},\{a,b,d\}\}$. Let $f:(U,\tau_R(X),\mu_R(X)) \to (V,\tau'_R(Y),\ \mu'_R(Y))$ be a function defined by $f(a)=d, f(b)=c, f(c)=b, f(d)=a$. Clearly f is micro α^*-homeomorphism. Let $g:(V,\tau'_R(Y),\ \mu'_R(Y)) \to (W,\tau''_R(Z),\ \mu''_R(Z))$ be a function defined by $g(a)=b, g(b)=a, g(c)=d, g(d)=c$. Clearly g is micro α^*-homeomorphism. Here f and g are micro α^*-homeomorphism. But $g\circ f:(U,\tau_R(X),\mu_R(X)) \to (W,\tau''_R(Z),\ \mu''_R(Z))$ is not micro α^*-homeomorphism. Since $(g\circ f)^{-1}\{a,c\} = g^{-1}(f^{-1}(\{a,c\}) = g^{-1}\{b,d\} = \{a,c\}$ is not micro α^*open in $(U,\tau_R(X),\mu_R(X))$. Therefore $(g\circ f)$ is not Micro α^*-homeomorphism.

Theorem3.17:

Let $f:(U,\tau_R(X),\mu_R(X)) \to (V,\tau'_R(Y),\mu'_R(Y))$ be bijective and micro α^*-continuous. Then the followings are equivalent:

(i) f is micro α^*-open map.

(ii) f is micro α^*-homeomorphism.

(iii) f is micro α^*-closed map.

Proof:

$\boldsymbol{(i) \Longrightarrow (ii)}$

Let f be micro α^*-open map. By hypothesis, f is micro α^*-homeomorphism.

$\boldsymbol{(ii) \Longrightarrow (iii)}$

Let f be micro α^*-homeomorphism. Let S be micro closed in $(U,\tau_R(X),\mu_R(X))$.Then U-S is micro open in $(U,\tau_R(X),\mu_R(X))$. Since f is micro α^*-homeomorphism, $f(U\text{-}S)$ is micro α^*-open in $(V,\tau'_R(Y),\mu'_R(Y))$ which implies $f(U)$-$f(S)$ is micro α^*-open in $(V,\tau'_R(Y),\mu'_R(Y))$ then V-$f(S)$ is micro α^*-open in $(V,\tau'_R(Y),\mu'_R(Y))$ which implies $f(S)$ is micro α^*-closed in $(V,\tau'_R(Y),\mu'_R(Y))$. Hence f is micro α^*-closed map.

$\boldsymbol{(iii) \Longrightarrow (i)}$

Let f be micro α^*- closed map. Let S be micro open in $(U,\tau_R(X),\mu_R(X))$. Then U-S is micro closed in $(U,\tau_R(X),\mu_R(X))$. Since f is micro α^*-closed, $f(U\text{-}S)$ is micro α^*-closed in $(V,\tau'_R(Y),\mu'_R(Y))$which implies $f(U)$-$f(S)$ is micro α^*-closed in $(V,\tau'_R(Y),\mu'_R(Y))$ which implies V-$f(S)$ is micro α^*-closed in $(V,\tau'_R(Y),\mu'_R(Y))$. which implies $f(S)$ is micro α^*-open in $(V,\tau'_R(Y),\mu'_R(Y))$. Hence f is micro α^*-open map.

Theorem3.18:

Let $f:(U,\tau_R(X),\mu_R(X)) \to (V,\tau'_R(Y),\mu'_R(Y))$ be bijection then the following statements are equivalent:

(i) f is micro α^*-homeomorphism

(ii) f is micro α^*-continuous and micro α^*-open map

(iii) f is micro α^*-continuous and micro α^*-closed map.

Proof:

$(\boldsymbol{i}) \Longrightarrow (\boldsymbol{ii})$

Let f be bijection and micro α^*-homeomorphism. Then f is micro α^*-continuous and micro α^*-open map.

$(\boldsymbol{ii}) \Longrightarrow (\boldsymbol{iii})$

Let f be micro α^*-continuous and micro α^*-open map. Let S be micro closed in $(U,\tau_R(X),\mu_R(X))$ then U-S is micro open in $(U,\tau_R(X),\mu_R(X))$. Since f is micro α^*-open map, $f(U\text{-}S)$ is micro α^*-open in $(V,\tau'_R(Y),\mu'_R(Y))$ which implies $V\text{-}f(S)$ is micro α^*-open in $(V,\tau'_R(Y),\mu'_R(Y))$ then $f(S)$ is micro α^*-closed in $(V,\tau'_R(Y),\mu'_R(Y))$. Thus f is micro α^*-closed map. Hence f is micro α^*-continuous and micro α^*-closed map.

$(\boldsymbol{iii}) \Longrightarrow (\boldsymbol{i})$

Let f be micro α^*-continuous and micro α^*-closed map. Let S be micro open in $(U,\tau_R(X),\mu_R(X))$ then U-S is micro closed in $(U,\tau_R(X),\mu_R(X))$. Since f is micro α^*-closed map then $f(U\text{-}S)$ is micro α^*-closed in $(V,\tau'_R(Y),\mu'_R(Y))$ which implies $V\text{-}f(S)$ is micro α^*-closed in $(V,\tau'_R(Y),\mu'_R(Y))$. Hence $f(S)$ is micro α^*-open in $(V,\tau'_R(Y),\mu'_R(Y))$. Therefore f is micro α^*-homeomorphism.

IV. CONCLUSION

In this paper, I have introduced Micro α^*- homeomorphism in Micro Topological Spaces with suitable examples. Then I have given some comparison theorems. Also I have discussed some of the properties of Micro α^*- homeomorphism. I hope that this paper is just beginning of a new structure. It will inspire many to contribute to the growth of micro topological spaces in the field of Mathematics. This shall be extended in the future research with some applications.

REFERENCES

[1] Dr. P. Anbarasi Rodrigo, A New class of micro open set in Micro Topology, Journal of Research in Applied Mathematics,Volume 8 ~ Issue 9 (2022) pp: 38-43

[2] R. Bhavani, On Strong Forms of Generalized Closed Sets in Micro Topological Spaces, Turkish Journal of Computer and Mathematics Education, Vol. 12, No.11(2021), 2772-2777.

[3] S.Chandrasekar, On micro topological spaces, Journal of New Theory,26(2019),23-31.

[4] S.Chandrasekar, G. Swathi, Micro- α-open Sets in Micro Topological Spaces, International Journal of Research in Advent Technology, Vol.6, No.10. October 2018, 2633-2637.

[5] Rangasamy Anandhi and Natraj Balamani, On Micro αg-irresolute maps in Micro Topological Spaces, Sahand Communications in Mathematical Analysis (SCMA) Vol. 22 No. 1 (2025), 75-92.

[6] Taha H. Jasim, Saja S. Mohsen, Kanayo S. Eke, On Micro-Generalized Closed Sets and Micro-Generalized Continuity in Micro Topological Spaces, European Journal of Pure and Applied Mathematics, Vol.14, no.4, 2021, 1507-1516.